3. Categorical Logic

CONTENTS

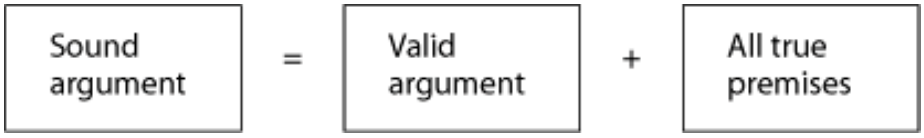
Focus

With diligent study of this guide, you will learn...	
Ideas	syllogism, validity, soundness
Skills	Determining validity in categorical arguments

3.1 Categorical Statements

3.1.1 Key Ideas / Terms

Key Ideas/Terms	Definition
syllogism	In general, a deductive argument consisting of exactly two premises and one conclusion.
categorical statement	A statement in which the members of one class are said to be included or excluded in/from another class.
categorical syllogism	Each statement in a ***categorical syllogism*** begins with a quantifier: *all*, *some*, or *no* (none) All B are C. major premise All A are B. minor premise Therefore, all A are C. conclusion
major premise	The statement in a syllogism that sets forth a general truth or principle.
minor premise	The statement in a syllogism that follows the first (major) premise.
quantifier	In a categorical syllogism, the words ***all***, ***some***, and ***no*** (none) specify how much of the Subject class is included or excluded from the Predicate class.
copula	In a categorical syllogism, the words ***are*** and ***are not*** are called the copula because they connect or couple the Subject term with the Predicate term.
A, E, I, and O propositions	The names of the four types of categorical propositions: **A** All S are P. **E** No S are P. **I** Some S are P. **O** Some S are not P.
valid deductive argument	An argument in which it is impossible for the conclusion to be false given that the premises are true. In these arguments, the conclusion follows with strict necessity from the premises because of the argument's *form*. *Any deductive argument having true premises and a false conclusion is necessarily invalid.*

Key Ideas/Terms	Definition
invalid deductive argument	A deductive argument in which it *is* possible for the conclusion to be false given that the premises are true. In these arguments the conclusion does not follow with strict necessity from the premises, even though it is claimed to.
sound argument	A deductive argument that is *valid* and has *all true premises*. Both conditions must be met for an argument to be sound; if either is missing the argument is unsound. A sound argument, therefore, is what is meant by a good, or successful, deductive argument in the fullest sense of the term.  It is not always possible to determine the soundness of a deductive argument. But that does not mean that soundness is unimportant in logic. It is crucially important that soundness be recognized as a criterion of evaluation that is distinct from validity.
unsound argument	A deductive argument that is invalid, has one or more false premises, or both.

3.1.2 Components of Categorical Statements

In a categorical syllogism, each of the three statements expresses a relationship between two categories or sets of entities—a ***S***ubject term and a ***P***redicate term. The standard-form of a categorical proposition is:

<quantifier> <*S*> <copula> <*P*>

Study the following syllogism and the related terminology. Assume that the term *Earthies* refers to the set of all inhabitants of Earth:

Categorical Statements	Argument Part	Quantifier	Subject Term	Copula	Predicate Term
All humans are Earthies.	*major* premise	All	humans	are	Earthies
All Americans are humans.	*minor* premise	All	Americans	are	humans
Therefore, all Americans are Earthies.	conclusion	All	Americans	are	Earthies

3.1.2 Substitution Instances

Every categorical syllogism is composed of three categorical statements or propositions that are in standard form. A categorical statement is said to be in standard form if and only if it is a substitution instance of one of these four *statement forms*:

Four Categorical Statement Forms	Form Identifier
All S are P.	**A**
No S are P.	**E**
Some S are P.	**I**
Some S are not P.	**O**

Note that the following argument is composed of three statements that are substitution instances of A-type statements:

All Humans are Earthies.
All Americans are Humans.
Therefore, all Americans are Earthies.

Also note that you can replace the category names with letters. By stripping out the details of this argument, and replacing the subject and predicate terms with letters, it is easy to discern the structure or form of the argument. For example, you could translate the syllogism above as:

All H are E.
All A are H.
Therefore, all A are E.

Next, note that you know it's impossible for the conclusion of this argument to be false if the premises are in fact true. This is called a *valid argument*. Also, you can see that every argument in this form will a valid argument. Hence every argument that is a substitution instance of this form, will be a valid argument.

3.1.3 Counterexample Method for Determining Validity

Logic is concerned with **argument form**. Logic is not concerned with the soundness of arguments because, in general, logic alone cannot determine whether the premises of an argument are true or false. In general, the grammatical structure of an English sentence mirrors its logical structure.

Now consider this argument form:

All A are B.
All C are B.
Therefore, all A are C.

Is this a valid argument form? Well, you can prove an argument form is invalid if you can create a substitution instance with actually true premises and a false conclusion. For example, consider this substitution instance of the form above:

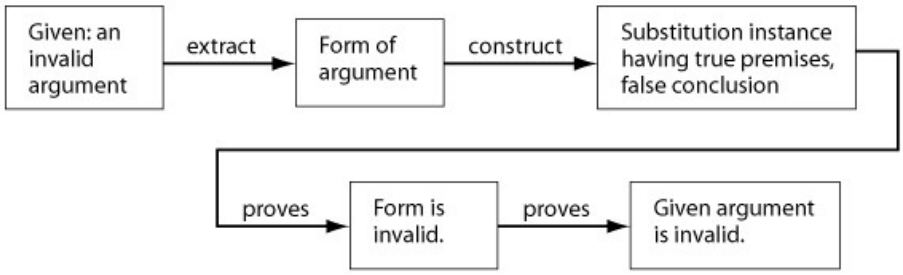
All cats are animals.
All ferrets are animals.
Therefore, all cats are ferrets.

Here we have true premises but a false conclusion. So, this substitution instance proves that this is not a valid argument form. Every invalid argument is worthless and should be discarded. If true premises don't support an inference to a true conclusion, the whole argument is useless.

Here is a simple procedure for proving the invalidity of any invalid argument:

1. Isolate the form of the argument.
2. Construct a substitution instance having true premises and a false conclusion.

This procedure proves the form invalid, which in turn proves the argument invalid.



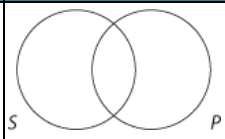
▶ Hurley, 1.4

it is useful to keep in mind the following set of terms: “cats,” “dogs,” “mammals,” “fish,” and “animals.” Most invalid syllogisms can be proven invalid by strategically selecting three of these terms and using them to construct a counterexample. Because everyone agrees about these terms, everyone will agree about the truth or falsity of the premises and conclusion of the counterexample.

3.1.4 Quantity, Quality, and Distribution

Quantity and *quality* are attributes of categorical statements. *Distribution* is an attribute of the terms (Subject and Predicate) in a categorical statement.

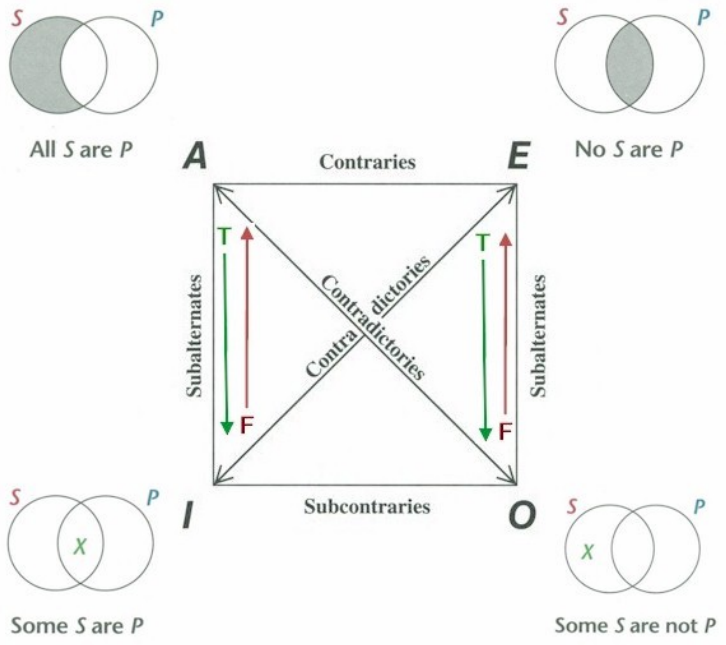
Key Ideas/Terms	Definition
quantity	The *quantity* of a categorical statement is either ***universal*** or ***particular***, depending on whether the statement makes a claim about every member or only some member of the class denoted by the subject term.
quality	The *quality* of a categorical statement is either ***affirmative*** or ***negative***, depending whether it affirms or denies class membership.
distribution	A term is said to be distributed if the statement makes an assertion about every member of the class denoted by the term. If a statement makes an assertion about every member of the S class, then S is distributed. So, universal (A and E) statements distribute their Subject terms. If a statement makes an assertion about every member of the P class, then P is distributed. Universal negative (E) statements, and particular negative (O) statements distribute their Predicate terms. Otherwise, S and P are undistributed.

Key Ideas/Terms	Definition
Venn diagram	 An arrangement of overlapping circles in which each circle represents the class denoted by a term in a categorical proposition. Every categorical proposition has exactly two terms (S and P), so the Venn diagram for a single categorical proposition consists of two overlapping circles. Each circle is labeled so that it represents one of the terms in the proposition. In general, the left-hand circle represents the subject (S) term, and the right-hand circle the predicate (P) term. If an area is ***shaded***, there are ***no items*** in it. If an area contains an "***x***" there is ***at least one item*** in it.

Statement Form	Identifier	Quantity	Quality	Terms Distributed
All S are P.	**A**	universal	affirmative	S
No S are P.	**E**	universal	negative	S and P
Some S are P.	**I**	particular	affirmative	Neither S nor P
Some S are not P.	**O**	particular	negative	P

3.1.5 The Traditional Square of Opposition

Study this diagram carefully. Note that for the Venn diagrams, a shaded area represents emptiness, and an "X" represents the existence of at least one entity.



Contradictory

Opposite truth values

Contrary

At least one is false (not both true).

Subcontrary

At least one is true (not both false).

Subalternation

Truth flows downward; falsity flows upward. For example, **all S are P** implies that **some S are P**. However, asserting that **some S are P** does not imply that **all S are P**.

3.1.6 Existential Fallacy

It is possible to interpret universal (**A** and **E**) statements in two different ways. Following one interpretation, an argument might be valid. But in the other interpretation, it might be invalid. Consider these two statements:

1. All Brad Pitt's movies are hits.
2. All unicorns are single-horned animals.

In ordinary discourse, the first statement implies that Brad Pitt has made some movies. That is to say, this statement has existential import—we are talking about things that actually exist. But with the second statement, there is merely a definition; there is no implication that unicorns actually exist.

So, should universal propositions be interpreted as implying that the things talked about actually exist? There are two different answers to this question.

▶ Aristotelian Standpoint

Aristotle took the position that only universal statements about *real* or *existing beings* have existential import.

Universal Statements	Existential Import?
All ferrets are mammals.	**Yes.** Implies the existence of ferrets.
No begonias are ferns.	**Yes.** Implies the existence of begonias.
All zombies are nocturnal.	**No.** Does not imply the existence of zombies.

▶ Boolean Standpoint

According to the theory of George Boole, a 19th century mathematician and philosopher who invented Boolean algebra, and with the further elaboration by John Venn (inventor of Venn diagrams), no universal propositions imply the existence of the things talked about.

Universal Statements	Existential Import?
All ferrets are mammals.	**No.** Does not imply the existence of ferrets.
No begonias are ferns.	**No.** Does not imply the existence of begonias.
All zombies are nocturnal.	**No.** Does not imply the existence of zombies.

From the Boolean standpoint, we commit the existential fallacy if we were to make this inference:

All A are B.
Therefore, some A are B.

The Aristotelian standpoint differs from the Boolean standpoint only regarding universal (**A** and **E**) statements. Both the Aristotelian and Boolean standpoints assert that particular (**I** and **O**) statements have existential import. From both standpoints, the statements: "Some fish are edible," and "Some fish are not edible," imply existence.

Let's compare the inferences that are valid and invalid from these two different viewpoints:

All ferrets are mammals. So, some ferrets are mammals.	**Boolean:** Invalid, existential fallacy **Aristotelian:** Valid
All zombies are nocturnal. So, some zombies are nocturnal.	**Boolean:** Invalid, existential fallacy **Aristotelian:** Invalid, existential fallacy

In general, the Boolean interpretation of universal statements makes logical analysis simpler. But the Aristotelian standpoint comports better with our ordinary usage of language and general intention to speak about real things in our arguments.

3.1.7 Translating Ordinary Language Statements

Ordinary Language	Translation to Categorical Form
Terms without nouns Eg: Some daisies are yellow.	Some daisies are yellow flowers.
Non-standard verbs Eg: All college students will become educated.	All college students are people who will become educated.
Singular propositions Eg: Socrates is mortal.	All people identical to Socrates are people who are mortal.
Spatial adverbs (where, wherever, anywhere, everywhere, nowhere) Eg: Nowhere on Mars are there any aliens.	Translate as *places*: No places on Mars are places where there are aliens.
Temporal adverbs (when, whenever, anytime, always, never) Eg: She always wears boots to class.	Translate as *times*: All times she goes to class are times she wears boots.
Pronouns (who, whoever, anyone, whatever, anything) Eg: Whoever is diligent will succeed.	Translate as *people* or *things*: All people who work hard are people who will succeed.
Unexpressed quantifiers (implied by context) Eg: Latvians are rapists and drug dealers.	Some means at least one: Some Latvians are rapists and drug dealers.
Non-standard quantifiers (few, anyone, many, several, not everyone) Eg: A few people are pick-pockets.	Some means at least one: Some people are pick-pockets.
Conditional statements Eg: If it's a rat then it's a mammal.	Term following the *if* (antecedent), is the subject in an **A** proposition: All rats are mammals.
Conditional statements with negated consequents Eg: If it's a cobra, it's not a mammal.	Conditional statement with a negated consequent is best translated as an **E** proposition: No cobras are mammals.
Conditional statements with *unless* Eg: Avocados are edible unless they are spoiled.	*Unless* means *not if*. The term following *if* (antecedent), is the subject in an **A** proposition: All unspoiled avocados are edible avocados.
Exclusive propositions (only, none but, none except, no...except) Eg: None but the brave deserve glory.	The term following the exclusive word is the predicate. All people who deserve glory are brave people.

Ordinary Language	Translation to Categorical Form
***The only* propositions** Eg: The only animals that live in my neighborhood are nocturnal.	The term following the only word is the subject in an **A** proposition:: All animals that live in my neighborhood are nocturnal.
Exceptive propositions ("All except S are P" and "All but S are P") Eg: All but students must report to the dean.	Exceptive propositions must be translated into two categorical propositions: No students are people who must report to the dean. All non-students are people who must report to the dean.
Note: Because an exceptive proposition requires translation into two categorical propositions, this kind of ordinary language statement cannot be used as a premise or conclusion in a categorical syllogism. They must be translated and evaluated in propositional logic (ICT Study Guide 4).	

3.1.8 Logically Equivalent Statement Forms

Conversion, obversion, and contraposition are operations that can be performed on a categorical proposition that may or may not result in the same meaning and truth value as the original statement.

Given Statement	Converse	Obverse	Contrapositive
A: All A are B	All B are A	No A are non-B	All non-B are non-A
E: No A are B	No B are A	All A are non-B	No non-B are non-A
I: Some A are B	Some B are A	Some A are not non-B	Some non-B are non-A
O: Some A are not B	Some B are not A	Some A are non-B	Some non-B are not non-A

For more information on logically equivalent statement forms, using conversion, obversion, and contraposition, refer to **Logic Ref**: *2.7 Logically Equivalent Statement Forms.*

3.2 Categorical Syllogisms

Key Ideas/Terms	Definition
standard form of a categorical statement	Where ***S*** denotes the subject term, and ***P*** denotes the predicate term, the standard-form of a categorical proposition is: **<quantifier> < *S* > <copula> < *P* >**
form of a syllogism	The *form* of a syllogism is determined by its *mood* and *figure*. After a categorical syllogism has been put into *standard form*, its validity or invalidity can be determined by merely inspecting its form.

Key Ideas/Terms	Definition
major term	Occurs in the major premise and is the predicate of the conclusion of a syllogism. For example, in this syllogism, **E** is the *major term*: All H are **E**. All A are H. Therefore, all A are **E**.
minor term	Occurs in the minor premise and is the subject of the conclusion of a syllogism. For example, in this syllogism, **A** is the *minor term*: All H are E. All **A** are H. Therefore, all **A** are E.
middle term	The "link" between the two premises. Occurs once in each premise, and not in the conclusion. For example, in this syllogism, **H** is the *middle term*: All **H** are E. All A are **H**. Therefore, all A are E. NOTE: The middle term must be distributed at least once in a valid syllogistic argument. In the argument above, the middle term is distributed in the major premise (which makes an assertion about every member of the class denoted by the term).
mood	Denoted by the letter names (A, E, I, O), of its constituent propositions. The letter for the major premise is listed first, then the letter for the minor premise, and finally the letter for the conclusion.
figure	Determined by the position of the occurrences of the **M**iddle term in the premises. There are four patterns or figures:

Figure 1	Figure 2	Figure 3	Figure 4
M P	P M	M P	P M
S M	S M	M S	M S
S P	S P	S P	S P

Study the following syllogism again, assuming that that the term *Earthies* refers to the set of all inhabitants of Earth:

Categorical Statements	Argument Part	Quantifier	Subject Term	Copula	Predicate Term
All humans are Earthies.	*major* premise	All	humans	are	Earthies
All Americans are humans.	*minor* premise	All	Americans	are	humans
Therefore, all Americans are Earthies.	conclusion	All	Americans	are	Earthies

3.3 Determining Validity of Categorical Syllogisms

Most people intuitively evaluate this argument as valid—that is, an argument in which it is impossible for the conclusion to be false if the premises are true. But some argument forms are more complex, and their validity is more difficult to discern.

We have already seen that the counterexample method can be used to evaluate the validity of an argument. We could also use the traditional square of opposition or Venn diagrams. But here are two methods that are straightforward and reliable.

3.3.1 Using Rules to Determine Validity

3.3.1 (1) Put the categorical syllogism in standard form.

A standard form categorical syllogism requires four conditions:

1. All three statements are standard form categorical statements.
2. The two occurrences of each term are identical.
3. Each term is used in the same sense throughout the argument (no equivocation).
4. The major premise is listed first, the minor premise second, and the conclusion last.

1. Quantifier	_____	copula	_____	Major premise (contains the *major term*)
2. Quantifier	_____	copula	_____	Minor premise (contains *minor term*)
3. Quantifier	(minor term)	copula	(major term)	Conclusion

3.3.1 (2) Check the quantity, quality, and term distribution

After a categorical syllogism has been put into *standard form*, its validity or invalidity can be checked by following the rules summarized in this checklist:

	Confirm that...	Fallacy
❑	1. The **middle** term is **distributed** at least once.	Undistributed middle
❑	2. If a term is **distributed** in the conclusion, then it must be **distributed** in the premise. (A term is said to be distributed if the statement makes an assertion about every member of the class denoted by the term.)	Illicit major, illicit minor
❑	3. There is **only one negative premise**. (2 negative premises are not allowed.)	Exclusive premises
❑	4. If there is a **negative premise**, there must be a **negative conclusion**; and if there is a **negative conclusion**, there must be a **negative premise**.	Affirmative conclusion from negative premise, negative conclusion from affirmative premises
❑	5. Adopting the Aristotelian standpoint, **if both premises are universal, the conclusion cannot be particular unless** terms denote actually existing things.	Existential fallacy

3.3.2 Using Look-up Tables to Determine Validity

3.3.2 (1) Put the categorical syllogism in standard form.

Refer to 3.3.1 (1) above.

3.3.2 (2) Identify the form of the syllogism.

After a categorical syllogism has been put into *standard form*, its validity or invalidity can be determined by merely inspecting its form (*mood* and *figure*).

In general, if a syllogism with a given form is a *valid* argument, then all syllogisms having that form are valid. And if a syllogism with a given form is an *invalid* argument, then all syllogisms having that form are invalid. So, in this second step, identify both the mood and figure of the syllogism in question.

▶ **Identify the *mood*** of the syllogism by assigning a statement ID (A, E, I, O) to each of the premises and conclusion.

For example, the *mood* of this argument is **EIO**:

E	No cyborgs are Martians.
I	Some Martians are vegetarians.
O	Therefore, some vegetarians are not cyborgs.

▶ **Identify the figure** of the syllogism:

a) Drop the quantifiers and copulas.

b) Determine the positions of the three terms in the syllogism:

S =the Subject of the conclusion (minor term)

P = the Predicate of the conclusion (major term)

M = the middle term (occurs once in each premise but not in the conclusion)

Figure 1	Figure 2	Figure 3	Figure 4
M P	P M	M P	P M
S M	S M	M S	M S
S P	S P	S P	S P

For example, the following syllogism has *figure* **4**:

E	No cyborgs are Martians.	**major term:** cyborgs
I	Some Martians are vegetarians.	**minor term:** vegetarians
O	Therefore, some vegetarians are not cyborgs.	

3.3.2 (3) Determine validity through look up.

Once you have determined the *mood* and *figure* of a syllogism, you can determine the validity of the argument's form by referring to the following look-up tables:

Unconditionally Valid Syllogistic Forms

Figure 1	Figure 2	Figure 3	Figure 4
M P S M S P	P M S M S P	M P M S S P	P M M S S P
AAA EAE AII EIO	EAE AEE EIO AOO	IAI AII OAO EIO	AEE IAI EIO

Conditionally Valid Syllogistic Forms

Figure 1	Figure 2	Figure 3	Figure 4	Required Condition
AAI EAO	AEO EAO		AEO	S exists
		AAI EAO	EAO	M exists
			AAI	P exists

A critical thinker uses reasoning to discover truth and prevent stereotyping.

> Self-reflection, also called introspection, is a means to observe and analyze oneself in order to grow as a person. That growth is the reason why it is so important to spend time in personal reflection. By understanding who you are now and who you'd like to become, you help identify the steps you need to take on that journey. Reflecting upon how you behave and what thoughts enter your mind in response to events in the world around you allows you to see what you need to work on. — A Conscious Rethink

Sharpen Your Critical Thinking

Identify at least one reality assumption behind each of these statements:

- Even if a product may not help me, at least it won't hurt me.
- When a food product is labeled as *All Natural Ingredients*, it means that the product is healthy and safe.
- If you travel far enough, you will fall off the end of the Earth.
- God has given us dominion over the Earth, therefore we can use the planet in whatever way we want.

PAUSE & REFLECT

A. Do I understand all the new concepts I have encountered so far?

B. What are my strengths or weaknesses in my critical thinking?

C. Has anything I have now learned about categorical arguments changed or affected my general disposition or any beliefs, values, perspectives, interests, or goals?

D. What difference could/might my knowledge and skills in critical thinking make for my family or community or country or planet? How could the world change because of my critical thinking mojo?"

3.4 Assessing My Critical Thinking

Self-reflection is a necessary habit for critical thinkers. To be a strong critical thinker means to habitually reflect on, and evaluate one's experience. The process of self-reflection can be envisioned as a continuous learning cycle grounded in a person's experience.

Exercise 3	
If a friend or fellow student is not available to help you with this exercise, simply imagine someone asking you to explain these ideas and answer these questions. ▶ If you are confident in the clarity, accuracy, and completeness of your explanations, continue forward on the path. Otherwise, go back and study the areas where you have stumbled, and then return to this exercise.	▪ **What is a categorical syllogism? What is an example of one?** ▪ **What are the four types or forms of categorical propositions?** ▪ **Do people always specify the quantifier when speaking categorically? Can this lead to problems?** ▪ **What is a valid argument? What is a sound argument?** ▪ **How does the counterexample method for determining validity work?** ▪ **Is there another method for determining the validity of a categorical syllogism?**

Quiet Reflection 3	
Self-reflection requires mental focus and personal honesty. At steps 2 and 3 especially, silence is very important. You must be able to hear your inner voice. Find a place that is quiet and comfortable. Turn off your phone and eliminate other distractions if possible.	
1. Observe/Study	▪ What public-interest issue would I like to investigate more at this point in my life?
2. Judge/Evaluate	▪ How often do I reflect on my ways of arguing? ▪ Do I believe that my public discourse can make a difference in the world? ▪ What is one of my strengths as a critical thinker? ▪ What is one of my weaknesses as a critical thinker?
3. Act/Decide	▪ What is a decision that I have made or need to make for my short-term flourishing? For my long-term flourishing? ▪ How could my commitment to always seek the truth affect my family, neighborhood, community, and the whole planet?