Study Guide 4

4. Propositional Logic

CONTENTS

Focus

With diligent study of this guide, you will learn...	
Ideas	valid and sound arguments, propositional logic, conditional statements, valid argument forms, *modus ponens, modus tollens,* disjunctive syllogism, invalid argument forms, truth tables
Skills	Converting ordinary language arguments into symbolic form, using truth tables to prove validity or invalidity, using the natural deduction method

Key Ideas / Terms

Key Ideas/Terms	Definition
conditional statements	Statements of the form: ***If p then q*** are called conditional statements. By themselves they are not arguments. They merely assert a conditional relation between the antecedent (***p***), and the consequent (***q***).
valid deductive argument	An argument in which it is impossible for the conclusion to be false given that the premises are true. In these arguments, the conclusion follows with strict necessity from the premises because of the argument's *form*. *Any deductive argument having true premises and a false conclusion is necessarily invalid.*
invalid deductive argument	A deductive argument in which it *is* possible for the conclusion to be false given that the premises are true. In these arguments the conclusion does not follow with strict necessity from the premises, even though it is claimed to.
sound argument	A deductive argument that is *valid* and has *all true premises*. Both conditions must be met for an argument to be sound; if either is missing the argument is unsound. A sound argument, therefore, is what is meant by a good, or successful, deductive argument in the fullest sense of the term. Sound argument = Valid argument + All true premises It is not always possible to determine the soundness of a deductive argument. But that does not mean that soundness is unimportant in logic. It is crucially important that soundness be recognized as a criterion of evaluation that is distinct from validity.
unsound argument	A deductive argument that is invalid, has one or more false premises, or both.

4.1 Propositional Logic

You have learned that categorical logic involves categorical statements that contain *quantifiers* (all, some, or none), and fundamental elements called *terms* (subject terms and predicate terms). But another kind of deductive reasoning uses whole statements (propositions) as the fundamental elements. In propositional logic, we use upper-case letters to represent whole statements, and a few symbols called *operators* to connect or relate these statements. For example, consider these two simple arguments:

	Argument	Form
1.	If the **S**pider is a black widow, then it is **D**angerous. The **S**pider is a black widow. Therefore, it is **D**angerous.	If **S** then **D**. **S**. / **D**.
2.	If **A**lphonse went to see *The Terminator*, then **P**ricilla went too. **A**lphonse went to see *The Terminator*. Therefore, **P**ricilla went too.	If **A** then **P**. **A**. / **P**

4.1.1 Statement Letters & Variables

Although the subject matter in these two arguments is different, and although we assigned different statement letters in each argument, it is easy to see that they have the same structure or form if we assign statement variables:

Statement Variable	Argument 1	Argument 2
p	the spider is a black widow	Alphonse went to see *The Terminator*
q	it is dangerous	Pricilla went too

Hence, the form of each of these arguments can be symbolized as:

So, both arguments 1 and 2 above have the same basic form:

If p then q
p
Therefore, q

4.1.2 Connectives

In both arguments 1 and 2 above, the first premise is a conditional statement (If...then). In the propositional arguments that follow, we use a set of simple symbols to represent the basic connectives or *logical operators* of ordinary language:

Symbol	Meaning	Example 2
p, q, r...	Statement variables	p > q
A, B, C...	Statement letters	S = The spider is a black widow.
~ (tilde)	**NOT** (negation)	~S = The spider is not a black widow. (It is not the case...) ~(S & D) | D & ~S | ~[(S & D) v (A > B)]
& (ampersand)	**AND** (conjunction)	S & D = The spider is a black widow and it is dangerous.

Symbol	Meaning	Example 2
v (vee)	**OR** (disjunction)	S v D = The spider is a black widow or it is dangerous.
> (right arrow)	**IF...THEN** (conditional	S > D = If The spider is a black widow, then it is dangerous.
(…) […] {…}	Scope markers	(p & q) v r = Either p and q, or r. (v is the main operator)
/ (slash)	**Therefore** (used to denote an argument's conclusion)	S > D S / D

Note that the NOT or negation operator (~) is always placed in front of the proposition it negates. And, unlike the other operators (&, v, and >), the ~ cannot be used to connect two propositions. But the tilde can immediately follow another operator, for example: D & ~S. If the scope of the negation is more than a simple statement, the scope is denoted by the use of parentheses, brackets, and braces, for example: ~[(S & D) v (A > B)].

4.1.3 Symbolizing Complex Statements

Simple (atomic) Statements

Ordinary Language Statement	Symbolization
Alphonse went to the opera.	A
Pricilla ate all the bonbons.	P
All humans make **M**istakes.	M
Socrates is mortal.	S

Complex (compound) Statements

Ordinary Language Statement	Symbolization
Socrates is not mortal.	~S
Alphonse went to the opera and **P**ricilla ate all the bonbons.	A & P
Either **P**ricilla ate all the bonbons or **A**lphonse went to the opera.	P v A
Either all humans make **M**istakes or else they don't.	M v ~M
If **P**ricilla ate all the bonbons, then **S**ocrates is mortal.	P > S
If all humans make **M**istakes and **A**lphonse went to the opera, then either **P**ricilla ate all the bonbons or **S**ocrates is mortal.	(M & A) > (P v S)

Note that the negation (r operator) of a simple statement is considered a compound statement since the operator alters the truth value of the simple statement.

4.2 Valid Argument Forms

In this section, you will study eight valid argument forms. Any argument that employs one or more of these forms will always have a true conclusion if the premises are true. Because all of these forms represent valid argument, they can be used as *rules of inference* for proving the validity of extended arguments.

4.2.1 *Modus Ponens* (MP)

p > q
p
/ q

Example Ordinary Language Arguments via MP	Symbolization
If Manuel is healthy, the Blazers will win.	M > B
Manuel is healthy.	M
Therefore, the Blazers will win.	/ B
If the Blazers win and the Weather permits, I will celebrate at the Lucky Lab Pub.	(B & W) > L
The Blazers win and the Weather permits.	B & W
So, I will celebrate at the Lucky Lab Pub.	L

Note that the statement variables *p* and *q* can represent simple as well as complex propositions. So, for example, this argument is also a case of *modus ponens*:

(B & W) > (L v Z)
B & W
/ L v Z

In the example above, there is a conditional statement, there is a premise that asserts the antecedent of the conditional, and there is a conclusion that asserts the consequent of the conditional: *modus ponens*.

Also note that simple statements can also be negative. For example, these are all cases of *modus ponens*:

~B > ~W	M > ~L	(B & W) > ~(L v Z)
~B	M	B & W
/ ~W	/ ~L	/ ~L v Z

Note, too, that the order of the premises does not matter, but that the conclusion is always stated last in a well-formed argument. For example:

B & W
(B & W) > (L v Z)
/ L v Z

Finally, please note that these clarifications apply to all the argument forms discussed here.

4.2.2 *Modus Tollens* (MT)

p > q
~q
/ ~p

Example Ordinary Language Arguments via MT	Symbolization
If Orwell is withholding evidence, then he is guilty.	O > G
Orwell is not guilty.	~G
Therefore, Orwell is not withholding evidence.	/ ~O
If I am not diligent, I will not succeed.	~D > ~S
I will succeed.	S
Thus, I am diligent.	/ D

In the second argument above, note that the denial of the consequent of the conditional statement could be symbolized as ~~S. But this negation of a negated statement is equivalent to the simple statement S. In the same way, the denial of the antecedent of the conditional statement (~~D), is equivalent to the simple statement D.

4.2.3 Disjunctive Syllogism (DS)

p v q		**p v q**
~p	OR	**~q**
/ q		**/ p**

Example Ordinary Language Arguments via DS	Symbolization
Either Bert is a stable genius or he is a monkey's uncle.	B v M
Bert is not a stable genius.	~B
It follows that Bert is a monkey's uncle.	/ M
Either Lolita will go to the conference or Joaquin will have a conniption.	L v J
Joaquin will not have a conniption.	~J
So, Lolita will go to the conference	/ L

PAUSE & REFLECT

A. Do I understand all the new concepts I have encountered so far?

B. What are my strengths or weaknesses in my critical thinking?

C. Has anything I have now learned about propositional logic changed or affected my general disposition or any beliefs, values, perspectives, interests, or goals?

4.2.4 Hypothetical Syllogism (HS)

p > q
q > r
/ p > r

Example Ordinary Language Arguments via HS	Symbolization
If taxes increase there will be massive strikes.	T > S
If there are massive strikes, then food rationing will follow.	S > R
Therefore, if taxes increase, food rationing will follow.	/ T > R
If taxes increase, the inflation rate will fall below two percent.	T > F
If the inflation rate does not fall below two percent, Wall Street will panic.	~F > P
But If the inflation rate does fall below two percent, there will be a market rebound.	F > M
Clearly, If taxes increase, then there will be a market rebound.	/ T > M

In the second argument above, note that the presence of an extraneous premise (second statement), does not invalidate this argument. This insight applies to all verbose arguments that include premises (statements) that do not contribute to the support of the conclusion.

4.2.5 Constructive Dilemma (CD)

p v q
p > r
q > s
/ r v s

Example Ordinary Language Arguments via CD	Symbolization
Either astrology is a science or it is mere flim-flam.	S v F
If it's a science, newspapers shouldn't publish horoscopes on the comics page.	S > ~C
And if it's mere flim-flam, then horoscopes shouldn't be published at all.	F > ~ P
Therefore, either they shouldn't publish horoscopes on the comics page or at all.	/ ~ C v ~P
Either I go or you go.	I v Y
If I go, I will be sleep-deprived.	I > S
If you go, you will be bored out of your mind.	Y > B
So, either I will be sleep-deprived or you will be bored out of your mind.	S/ S v B

4.2.6 Conjunction (Conj)

p		**p**
q	OR	**q**
/ p & q		**/ q & p**

Example Ordinary Language Argument via Conj	Symbolization
The roses are dying.	R
The pumpkins are ripening.	P
Therefore, the roses are dying and the pumpkins are ripening.	/ R & P

Note that this form of inference makes it explicit that it is valid to join two separate statements to form a single conjunctive statement.

4.2.7 Simplification (Simp)

p & q	OR	**p & q**
/ p		**/ q**

Example Ordinary Language Argument via Simp	Symbolization
Rosalie is a college student and a mother.	C & M
Rosalie is a mother.	/ M

Like Conjunction, an inference through Simplification is obvious and usually taken for granted.

4.2.8 Addition (Add)

p
/ p v q

Example Ordinary Language Arguments via Add	Symbolization
Imelda wears expensive emeralds.	I
Imelda wears expensive emeralds or the coffee is bitter.	/ I v C

This may seem like a strange way of reasoning, but it's valid. Consider that if *p* is true, then certainly, *p* or anything else will be a true statement.

4.3 Invalid Argument Forms

Again, consider this categorical syllogism:

> All A are B.
> All C are B.
> Therefore, all A are C.

You have already seen that this is not a valid form for a categorical syllogism. In this case, the middle term (B**)** is undistributed. Moreover, there is no figure that matches the figure of this syllogism. The form of this syllogism is defective (a formal fallacy), and every syllogism that has this form is invalid. And just as categorical logic has invalid argument forms, so too with arguments using propositional logic. Here, we will consider two invalid argument forms that involve *modus ponens* and *modus tollens*.

4.3.1 Affirming the Consequent - *Invalid Argument Form*

p > q
q
/ p

This formal fallacy is a corruption of the *modus ponens* form. It involves a conditional premise, a second premise that affirms the consequent, and a conclusion affirming the antecedent of the conditional premise.

Example Ordinary Language Argument -INVALID	Symbolization
If Alexander the Great was fatally wounded in a plane crash, then he is dead.	A > D
Alexander the Great is dead.	D
Therefore, Alexander the Great was fatally wounded in a plane crash.	/ A

In the example above, we see that the argument has true premises and a false conclusion—clearly an invalid argument. Every argument in this form is invalid!

4.3.2 Denying the Antecedent - *Invalid Argument Form*

p > q
~p
/ ~q

This formal fallacy is a corruption of the *modus tollens* form. It involves a conditional premise, a second premise that negates the antecedent, and a conclusion that negates the consequent of the conditional premise.

Example Ordinary Language Arguments via MP	Symbolization
If Alexander the Great was fatally wounded in a plane crash, then he is dead.	A > D
Alexander the Great was not fatally wounded in a plane crash.	~A
Therefore, Alexander the Great is not dead.	/ ~D

Again, in the example above, we see that the argument has true premises and a false conclusion. Every argument in this form is invalid!

4.4 Proving the Invalidity of Complex Arguments

4.4.1 Truth Tables

Key Ideas/Terms	Definition
truth value	Every simple proposition (***p***), has a *truth value* of either true or false (T or F). The truth value of a compound proposition (a statement that includes one or more logical operators), is a function of the truth values of its components. So, if the truth values of the components are known, then the truth value of the compound proposition can be determined from the definitions of the logical operators.
main operator	Every well-formed compound proposition, has only one main operator. Accordingly, the truth value of a well-formed compound proposition is the truth value of its main operator.
truth table	A *truth table* is an arrangement of truth values that shows, for every possible case, how the truth value of a compound proposition is determined by the truth values of its simple components. Each line in a truth table represents one possible arrangement of truth values.

To prove that an argument is valid or invalid, we can use truth tables. Carefully study the three simple truth tables that follow:

Truth Table for *Negation* ("~" operator definition)

Note that negating a statement (*p*), reverses its original truth value:

p	*~p*
T	F
F	T

Truth Table for *Conjunction* ("&" operator definition)

Note that the only case in which a conjunctive statement is true is when both of the *conjuncts* (*p, q*) are true:

p	*q*	*p & q*
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table for *Disjunction* ("v" operator definition)

Note that the only case in which a disjunctive statement is false is when both of the *disjuncts* (*p, q*) are false:

p	*q*	*p v q*
T	T	T
T	F	T
F	T	T
F	F	F

For more information on truth tables for the logical connectives (operators), refer to **Logic Ref**: *3.3 Truth Tables for Logical Operators.*

4.4.2 Example Truth Table for a Simple Argument

Here is a symbolized argument with three premises and a conclusion of P:

~ P v Q
~Q v R
~R
/ P

Study how this argument is laid out in its truth table:

P	Q	R	~P	v	Q	~Q	v	R	~R	/P
T	T	T	FT	T	T	FT	T	T	FT	T
T	T	F	FT	T	T	FT	F	F	TF	T
T	F	T	FT	F	F	TF	T	T	FT	T
T	F	F	FT	F	F	TF	T	F	TF	T
F	T	T	TF	T	T	FT	T	T	FT	F
F	T	F	TF	T	T	FT	F	F	TF	F
F	F	T	TF	T	F	TF	T	T	FT	F
F	F	F	TF	**T**	F	TF	**T**	F	**TF**	**F**

Please note that in the last row, the right-most column for the conclusion (/P), shows an F, and the main operators for the three premises show all Ts. This directly demonstrates that this argument is invalid since it produces a false conclusion when all the premises are true. Hence, when an argument is laid out in a truth table, if any row shows an F in the conclusion column when the values for the main operators are all T in that row, then the argument is invalid. A valid argument will never produce a false conclusion if the premises are in fact true.

For more information on using truth tables, refer to **Logic Ref**: *3 Propositional Logic*.

4.5 Proving Validity with Natural Deduction

Natural deduction is a method for determining the conclusion of a valid argument using the symbolism of propositional logic and the eight valid argument forms described in section 4.2 above and summarized at the right.

These eight valid forms can be considered as *rules of inference* that govern each step in a natural deduction proof. This method is called *natural deduction* because it is similar to the natural way in which humans reach conclusions through step-wise reasoning.

Modus Ponens (MP) p > q p / q	**Constructive Dilemma (CD)** p v q p > r q > s / r v s
Modus Tollens (MT) p > q ~q / ~p	**Conjunction (Conj)** p q / p & q OR p q / q & p
Disjunctive Syllogism (DS) p v q ~p / q OR p v q ~q / p	**Simplification (Simp)** p & q / p OR p & q / q
Hypothetical Syllogism (HS) p > q q > r / p > r	**Addition (Add)** p / p v q

Note that both truth table analysis and natural deduction can be used to prove the validity of a symbolized argument. However, **only truth tables can be used to prove invalidity**.

Now,consider this argument and study how it is comprised of three simple statements:

> If OxyContin is addictive, then either the pharmaceutical industry has stopped making it or thousands of people are becoming addicted to it. The pharmaceutical industry has not stopped making OxyContin. Certainly, OxyContin is addictive. Therefore, thousands of people are becoming addicted to it.

First, assign statement letters to represent the argument's statements:

Statement Key	
O	OxyContin is addictive.
P	The pharmaceutical industry has stopped making OxyContin.
T	Thousands of people are becoming addicted to OxyContin

Next, symbolize the premises on numbered lines (1-3), and examine the forms of the symbolic statements for opportunities to apply one of the eight inference rules.

4.5.1 Simple Proof Using Natural Deduction

1. O > (P v T)	premise 1	If OxyContin is addictive, then either the pharmaceutical industry has stopped making it or thousands of people are becoming addicted to it.
2. ~P	premise 2	The pharmaceutical industry has not stopped making OxyContin.
3. O	premise 3	(Certainly) OxyContin is addictive.
4. P v T	1, 3, MP	*modus ponens* - lines 1 and 3
5. T	2, 4 DS	disjunctive syllogism - lines 2 and 4 Therefore, thousands of people are becoming addicted to OxyContin.

In this case, it is easy to see that premise 3 (O) is the antecedent of the conditional statement on line one. So, by applying *modus ponens* (MP) to lines 1 and 3, we derive line 4 (P v T). And since one of the disjuncts on line four is negated by the premise on line 2, we can conclude to T on line five using the disjunctive syllogism (DS) rule. By deriving statement T, we have demonstrated that this is a valid argument.

Sharpen Your Critical Thinking

1. Compose a simple argument in ordinary language.
2. Construct a Statement Key for the statements in your argument.
3. Symbolize all the premises of your argument.
4. Either construct a truth table to verify the validity of your symbolized argument, or use natural deduction to derive your conclusion and prove the argument's validity.

4.6 Assessing My Critical Thinking

Exercise 4	
If a friend or fellow student is not available to help you with this exercise, simply imagine someone asking you to explain these ideas and answer these questions. ▶ If you are confident in the clarity, accuracy, and completeness of your explanations, continue forward on the path. Otherwise, go back and study the areas where you have stumbled, and then return to this exercise.	▪ **What is a statement or proposition?** ▪ **What is the difference between a statement variable and a statement letter?** ▪ **What is a valid argument? What is a sound argument?** ▪ **Name two or three valid argument form in propositional logic.** ▪ **What do the symbols: ~ , &, v, and > mean?** ▪ **How does the truth table method for determining validity work?** ▪ **Are there other methods for determining the validity of arguments in propositional logic?**

Quiet Reflection 4	
Self-reflection requires mental focus and personal honesty. At steps 2 and 3 especially, silence is very important. You must be able to hear your inner voice. Find a place that is quiet and comfortable. Turn off your phone and eliminate other distractions if possible.	
1. Observe/Study	▪ Begin to do library and online research for learning more about the issue or problem you identified in Worksheet 1 for your ICT Letter.
2. Judge/Evaluate	▪ What are the main areas or topics for my research? ▪ How do I find reliable sources for my research? ▪ How can I leverage one or more of my strengths as a critical thinker? ▪ How can I address my weaknesses as a critical thinker?
3. Act/Decide	▪ Am I collecting various opinions or viewpoints in my research for the ICT Letter? ▪ Build one or more arguments in support of the position you are taking on the issue covered in your ICT Letter. Symbolize them and use the eight rules of inference to demonstrate that they are valid arguments. ▪ Continue to reflect on how your commitment to always seek the truth affects your family, neighborhood, community, and the whole planet.