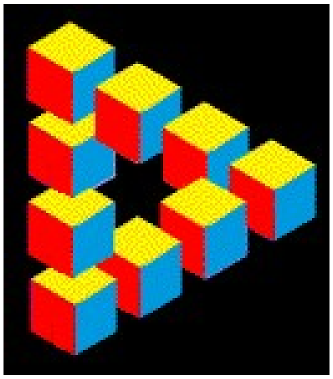


Quick Logic Guide

Portland Community College | 2018 Edition
PHL 221 | Symbolic Logic

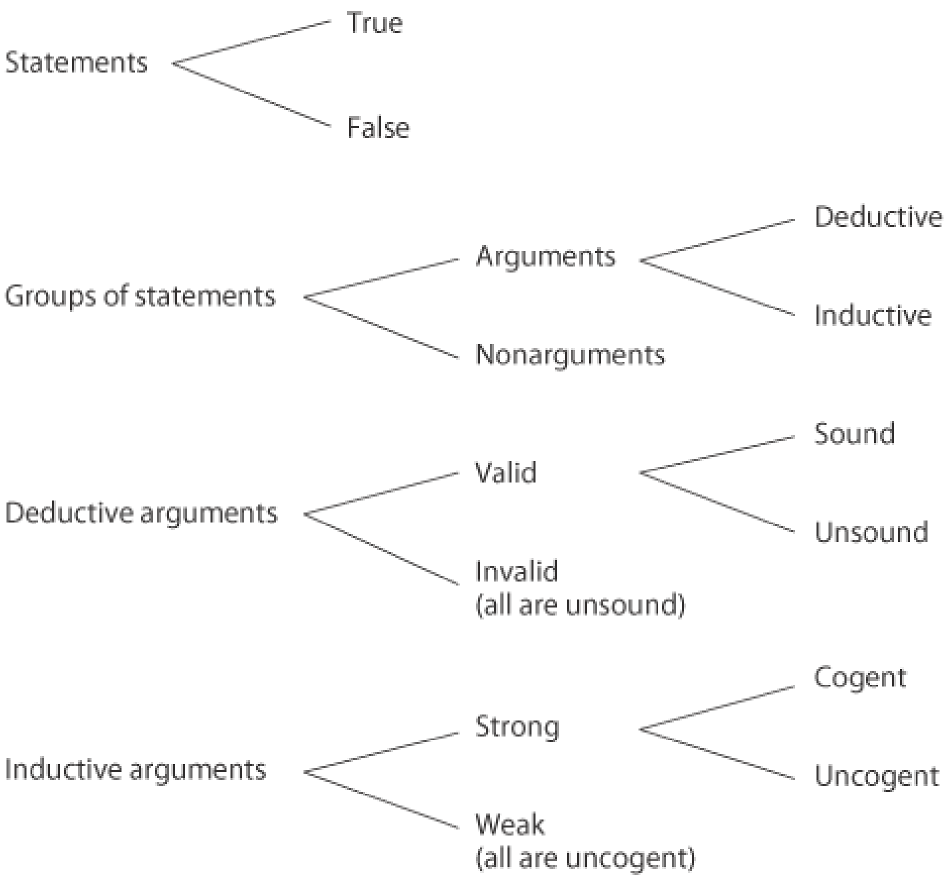
▶ This guide is a compact summary of the key concepts and methods in introductory deductive logic. It covers three symbolic languages: Categorical Logic, Propositional (Statement) Logic, and Predicate Logic.

▶ The terminology, symbols, definitions, and procedures in this guide are typically based on those used in *A Concise Introduction to Logic: An Emphasis on Modern Formal Logic,* 13th ed., Hurley & Watson, Cengage Learning, 2018.

1. Logic Overview

logic	The organized body of knowledge, or science, that evaluates *arguments*. As the science that evaluates arguments, the aim of logic is to develop methods and techniques that enable us to distinguish good arguments from bad ones.
first-order logic	A formal system used in mathematics, philosophy, linguistics, and computer science. It is also known as *first-order predicate calculus*, and *predicate logic*. Predicate logic (see section 4), is called 'first-order' because it allows quantifiers to range over objects (terms) but not over properties, relations, or functions applied to those objects. • In contrast to *propositional logic* (see section 3), use of truth tables does not work for finding valid sentential formulas in first-order predicate logic because its truth tables are infinite. However, *Gödel's completeness theorem* opens a way to determine validity, namely by proof. • No first-order theory has the strength to describe uniquely a structure with an infinite domain, such as the natural numbers or the real line. A uniquely describing, axiom system for such a structure can be obtained in stronger logics such as *second-order logic.*
modal logic	A modal is an expression (like 'necessarily' or 'possibly') that is used to qualify the truth of a judgment. Modal logic is, strictly speaking, the study of the deductive behavior of the expressions 'it is necessary that' and 'it is possible that'. • The term 'modal logic' may be used more broadly for a family of related systems. These include logics for belief, for tense and other temporal expressions, for the deontic (moral) expressions such as 'it is obligatory that' and 'it is permitted that', and many others. • An understanding of modal logic is particularly valuable in the formal analysis of philosophical argument, where expressions from the modal family are both common and confusing. Modal logic also has important applications in computer science.
benefits of logic	Among the benefits to be expected from the study of logic is an increase in confidence that we are making sense when we criticize the arguments of others and when we advance arguments of our own. *Once master the machinery of Symbolic Logic, and you have a mental occupation always at hand, of absorbing interest, and one that will be of real use to you in any subject you may take up. It will give you clearness of thought - the ability to see your way through a puzzle - the habit of arranging your ideas in an orderly and get-at-able form - and, more valuable than all, the power to detect fallacies, and to tear to pieces the flimsy illogical arguments, which you will so continually encounter in books, in newspapers, in speeches, and even in sermons, and which so easily delude those who have never taken the trouble to master this fascinating Art.* — Lewis Carroll

1.1 Basic Terminology Map



▶ Hurley, 1.4

1.2 Argument Basics

argument	A group of statements, one or more of which (the premises) are claimed to provide support for, or reasons to believe, one of the others (the conclusion). Every argument may be placed in either of two basic groups: those in which the premises really do support the conclusion (good arguments), and those in which they do not, even though they are claimed to (bad arguments).
statement	A sentence that is either true or false, typically a declarative sentence or a sentence component that could stand as a declarative sentence. The following sentences are statements: Chocolate truffles are loaded with calories. Melatonin helps relieve jet lag. Political candidates always tell the complete truth. No wives ever cheat on their husbands. Tiger Woods plays golf and Maria Sharapova plays tennis.
proposition	In the narrow sense, the meaning or information content of a statement. In this guide, “proposition” and “statement” are used synonymously.
truth values	Truth (T) and falsity (F) are called the two possible truth values of a statement.
premise(s)	The statement(s) that set forth the reasons or evidence for the conclusion.
conclusion	The statement that is claimed to follow from (supported by or inferred from) the premises.
inference	In the narrow sense of the term, is the reasoning process expressed by an argument. In the broad sense of the term, “inference” is used interchangeably with “argument.”
deductive argument	An argument incorporating the claim that it is *impossible* for the conclusion to be false given that the premises are true.
inductive argument	An argument incorporating the claim that it is *improbable* that the conclusion be false given that the premises are true.

1.2.1 Conclusion Indicators

Any statement following one of these indicators can usually be identified as a ***conclusion***:

therefore	**accordingly**	**entails that**
wherefore	**we may conclude**	**hence**
thus	**it must be that**	**it follows that**
consequently	**for this reason**	**implies that**
we may infer	**so**	**as a result**

1.2.2 Premise Indicators

Any statement following one of these indicators can usually be identified as a ***premise***:

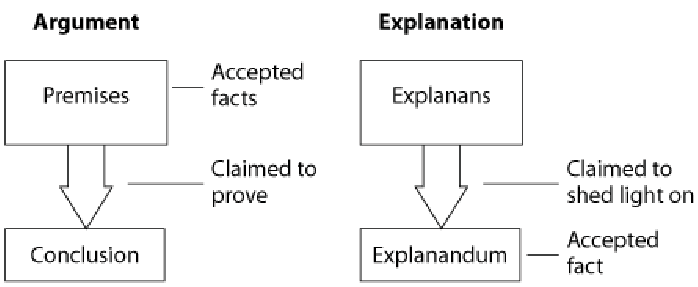
since	**in that**	**seeing that**
as indicated by	**may be inferred from**	**for the reason that**
because	**as**	**in as much as**
for	**given that**	**owing to**

1.2.3 Non Arguments

simple non-inferential passages	Passages (texts) that lack a claim that anything is being proved. Such passages contain statements that could be premises or conclusions (or both), but what is missing is a claim that any potential premise supports a conclusion or that any potential conclusion is supported by premises. Passages of this sort include: • warnings • pieces of advice • statements of belief or opinion • loosely associated statements • reports
expository passage	A kind of discourse that begins with a topic sentence followed by one or more sentences that develop the topic sentence. If the objective is not to prove the topic sentence but only to expand it or elaborate it, then there is no argument
illustration	An expression involving one or more examples that is intended to show what something means or how it is done. Often confused with arguments because many illustrations contain indicator words such as "thus."
explanation	An expression that purports to shed light on some event or phenomenon. The event or phenomenon in question is usually accepted as a matter of fact. Every explanation is composed of two distinct components: the ***explanandum*** and ***explanans***. The ***explanandum*** is the statement that describes the event or phenomenon to be explained, and the ***explanans*** is the statement or group of statements that purports to do the explaining.

1.2.4 Comparing Arguments and Explanations

An ***argument*** intends to establish *that* some state of affairs is the case. An ***explanation*** intends to establish *why* some state of affairs is the case:



1.2.5 Conditional Statements

conditional statement	an "if… then…" statement; for example: • If professional football games incite violence in the home, then the widespread approval given to this sport should be reconsidered. • If Roger Federer has won more Grand Slams than any other contender, then he rightfully deserves the title of world's greatest tennis player. Some conditional statements are similar to arguments in that they express the outcome of a reasoning process. As such, they may be said to have a certain inferential content. Conditional statements are especially important in logic (and many other fields) because they express the relationship between ***necessary*** and ***sufficient conditions***.
antecedent	The component statement immediately following the "if"

consequent	The component statement immediately following the "then" The link between the ***antecedent*** and ***consequent*** resembles the inferential link between the ***premises*** and ***conclusion*** of an argument. Yet there is a difference because the premises of an argument are claimed to be true, whereas no such claim is made for the antecedent of a conditional statement. Accordingly, conditional statements are not arguments.
necessary condition	B is said to be a ***necessary condition*** for A whenever A cannot occur without the occurrence of B . Thus, being an animal is a necessary condition for being a dog.
sufficient condition	A is said to be a ***sufficient condition*** for B whenever the occurrence of A is all that is needed for the occurrence of B. For example, being a dog is a ***sufficient condition*** for being an animal.

The relation between conditional statements and arguments can be summarized:

1. A single conditional statement is not an argument.
2. A conditional statement may serve as either the premise or the conclusion (or both) of an argument.
3. The inferential content of a conditional statement may be re-expressed to form an argument.

1.3 Argument Forms

1.3.1 Deductive Argument Forms

Argument based on mathematics	An argument based on mathematics is an argument in which the conclusion depends on some purely arithmetic or geometric computation or measurement. For example, a shopper might place two apples and three oranges into a paper bag and then conclude that the bag contains five pieces of fruit. Or a surveyor might measure a square piece of land and, after determining that it is on each side, conclude that it contains . Since all arguments in pure mathematics are deductive, we can usually consider arguments that depend on mathematics to be deductive as well. However, arguments that depend on statistics are a noteworthy exception and are usually best interpreted as inductive.
argument from definition	an argument in which the conclusion is claimed to depend merely on the definition of some word or phrase used in the premise or conclusion. For example, someone might argue that because Claudia is mendacious, it follows that she tells lies, or that because a certain paragraph is prolix, it follows that it is excessively wordy. These arguments are deductive because their conclusions follow with necessity from the definitions of "mendacious" and "prolix."
syllogism	In general, an argument consisting of exactly two premises and one conclusion.
categorical syllogism	Each statement in a ***categorical syllogism*** begins with one of the words "all," "no," or "some." **All As are Bs.** **All Bs are Cs** **Therefore, all As are Cs.**
hypothetical syllogism	A syllogism having a ***conditional*** ("if… then") statement for one or both of its premises. **If p then q.** **p.** **Therefore, q.**
disjunctive syllogism	A syllogism having a disjunctive ("either… or…") statement. **Either p or q.** **Not p.** **Therefore, q.**

1.3.2 Inductive Argument Forms

prediction	An argument that proceeds from our knowledge of the past to a claim about the future. Nearly everyone realizes that the future cannot be known with certainty; thus, whenever an argument makes a prediction about the future, one is usually justified in considering the argument inductive.
argument from analogy	An argument that depends on the existence of an analogy, or similarity, between two things or states of affairs. Because of the existence of this analogy, a certain condition that affects the better-known thing or situation is concluded to affect the similar, lesser-known thing or situation. The argument depends on the existence of a similarity, or analogy, between the two two things or states of affairs. The certitude attending such an inference is probabilistic at best.
generalization	An argument that proceeds from the knowledge of a selected sample to some claim about the whole group. Because the members of the sample have a certain characteristic, it is argued that all the members of the group have that same characteristic. Note the use of statistical samples in inductive argumentation.
argument from authority	An argument that concludes something is true because a presumed expert or witness has said that it is. Because authorities or experts can be either mistaken or lying, such arguments are essentially probabilistic.
argument based on signs	An argument that proceeds from the knowledge of a sign to a claim about the thing or situation that the sign symbolizes. The word "sign," as it is used here, means any kind of message (usually visual) produced by an intelligent being. Because signs can be misplaced or in error, conclusions based on them are only probable.
causal inference	An argument that proceeds from knowledge of a cause to a claim about an effect, or, conversely, from knowledge of an effect to a claim about a cause. Because specific instances of cause and effect can never be known with absolute certainty, one may usually interpret such arguments as inductive.
scientific arguments	Arguments that occur in science can be either inductive or deductive, depending on the circumstances. In general, arguments aimed at the *discovery* of a law of nature are usually considered inductive.

1.4 Evaluating Arguments

Regardless of the type of argument, whether deductive or inductive, the evaluation of any argument involves answering two distinct questions:

1. Do the premises support the conclusion?
2. Are all the premises true?

The answer to the first question is the more important one, because if the premises fail to support the conclusion (that is, if the reasoning is bad), the argument is worthless.

1.4.1 Evaluating Deductive Arguments

valid deductive argument	An argument in which it is impossible for the conclusion to be false given that the premises are true. In these arguments the conclusion follows with strict necessity from the premises.
invalid deductive argument	A deductive argument in which it *is* possible for the conclusion to be false given that the premises are true. In these arguments the conclusion does not follow with strict necessity from the premises, even though it is claimed to.
test for validity	To test an argument for validity we begin by assuming that all the premises are true, and then we determine if it is possible, in light of that assumption, for the conclusion to be false.
sound argument	A deductive argument that is *valid* and has *all true premises*. Both conditions must be met for an argument to be sound; if either is missing the argument is unsound. A sound argument, therefore, is what is meant by a good, or successful, deductive argument in the fullest sense of the term. Sound argument = Valid argument + All true premises It is not always possible to determine the soundness of a deductive argument. But that does not mean that soundness is unimportant in logic. It is crucially important that soundness be recognized as a criterion of evaluation that is distinct from validity.
unsound argument	A deductive argument that is invalid, has one or more false premises, or both.

	Valid	Invalid
True premises	All flowers are plants. All daisies are flowers.	All flowers are plants. All daisies are plants.
True conclusion	Therefore, all daisies are plants. **[sound]**	Therefore, all daisies are flowers. **[unsound]**
True premises	**None exist**	All roses are flowers. All daisies are flowers.
False conclusion		Therefore, all daisies are roses. **[unsound]**
False premises	All flowers are dogs. All poodles are flowers.	All dogs are flowers. All poodles are flowers.
True conclusion	Therefore, all poodles are dogs. **[unsound]**	Therefore, all poodles are dogs. **[unsound]**
False premises	All flowers are dogs. All tigers are flowers.	All roses are cats. All daisies are cats.
False conclusion	Therefore, all tigers are dogs. **[unsound]**	Therefore, all daisies are roses. **[unsound]**

Premises	Conclusion	Validity
T	**T**	**?**
T	**F**	**Invalid**
F	**T**	**?**
F	**F**	**?**

Merely knowing the truth or falsity of the premises and conclusion in a deductive argument tells us nothing about validity except in the one case of true premises and false conclusion. *Any deductive argument having true premises and a false conclusion is necessarily invalid.*

1.4.2 Evaluating Inductive Arguments

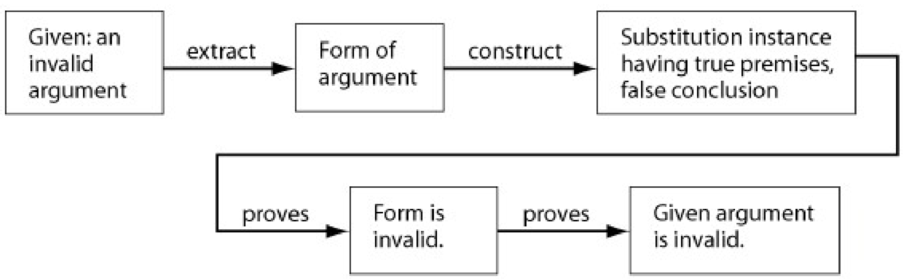
strong inductive argument	An inductive argument in which it is *improbable* that the conclusion be false given that the premises are true. In such arguments, the conclusion does in fact follow probably from the premises.
weak inductive argument	An inductive argument in which the conclusion *does not follow probably* from the premises, even though it is claimed to.
uniformity of nature	All inductive arguments depend on the principle that the future tends to replicate the past, and regularities that prevail in one spatial region tend to prevail in other regions. Good inductive arguments are those that accord with the uniformity of nature. They have conclusions that we naturally expect to turn out true.
test for strength	To test an inductive argument for strength we begin by assuming that all the premises are true, and then we determine whether, based on that assumption, the conclusion is probably true. This determination is accomplished by linking up the premises with regularities that exist in our experiential background. All of these regularities are instances of the uniformity of nature.
cogent argument	A is an inductive argument that is *strong* and has *all true premises*. Also, the premises must be true in the sense of meeting the *total evidence requirement*. If any one of these conditions is missing, the argument is *uncogent*. Cogent argument = Strong argument + All true premises A cogent argument is the inductive analogue of a sound deductive argument and is what is meant by a good, or successful, inductive argument without qualification.
uncogent argument	An inductive argument that is weak, has one or more false premises, fails to meet the total evidence requirement, or any combination of these.

	Strong	Weak
True premise	Every previous U.S. president was older than .	A few U.S. presidents were lawyers.
Probably true conclusion	Therefore, probably the next U.S. president will be older than . **[cogent]**	Therefore, probably the next U.S. president will be older than . **[uncogent]**
True premise	None exist	A few U.S. presidents were unmarried.
Probably false conclusion		Therefore, probably the next U.S. president will be unmarried. **[uncogent]**
False premise	Every previous U.S. president was a TV debater.	A few U.S. presidents were dentists.
Probably true conclusion	Therefore, probably the next U.S. president will be a TV debater. **[uncogent]**	Therefore, probably the next U.S. president will be a TV debater. **[uncogent]**
False premise	Every previous U.S. president died in office.	A few U.S. presidents were dentists.
Probably false conclusion	Therefore, probably the next U.S. president will die in office. **[uncogent]**	Therefore, probably the next U.S. president will be a dentist. **[uncogent]**

Premises	Conclusion	Strength
T	*probably* **T**	**?**
T	*probably* **F**	**Weak**
F	*probably* **T**	**?**
F	*probably* **F**	**?**

Merely knowing the truth conditions of the premises and conclusion tells us nothing about the strength of an argument except in the one case of true premises and probably false conclusion. *Any inductive argument having true premises and a probably false conclusion is weak.*

1.5 Proving Invalidity

argument form	An arrangement of letters (for example S, E, and B) and words (in this case "all" and "are") such that the uniform substitution of words or phrases in the place of the letters results in an argument. For example: All S are E. All B are S. All B are E.
substitution instance	For the form above, the words or phrases being substituted must refer to groups of things. Thus, if we substitute "sporting events," "engaging pastimes," and "baseball games" in the place of S, E, and B, respectively, we obtain the following argument: All sporting events are engaging pastimes. All baseball games are sporting events. All baseball games are engaging pastimes. The argument above is called a *substitution instance* of the argument form: All S are E. All B are S. All B are E. Every substitution instance of a valid form is a valid argument. But, not every substitution instance of an invalid form is an invalid argument.
counterexample method	A procedure for proving the invalidity of any invalid argument: 1. Isolate the form of the argument. 2. Construct a substitution instance having true premises and a false conclusion. This procedure proves the form invalid, which in turn proves the argument invalid.  ▶ Hurley, 1.4 it is useful to keep in mind the following set of terms: "cats," "dogs," "mammals," "fish," and "animals." Most invalid syllogisms can be proven invalid by strategically selecting three of these terms and using them to construct a counterexample. Because everyone agrees about these terms, everyone will agree about the truth or falsity of the premises and conclusion of the counterexample.

1.6 Analyzing Extended Arguments

procedure for analyzing extended arguments

1. Eliminate extraneous textual material. (Extended arguments are often mixed together with fragments of reports, pieces of expository writing, illustrations, explanations, and statements of opinion.)
2. Assign numbers to all *statements*.
3. Identify premises and conclusions using arrows to represent the inferential links.
4. Identify subarguments and separate strands of argumentation that lead to separate conclusions.

vertical pattern

A distinct pattern of argumentation in some extended arguments. The *vertical pattern* consists of a series of arguments in which a conclusion of a logically prior argument becomes a premise of a subsequent argument. For example:

①The selling of human organs, such as hearts, kidneys, and corneas, should be outlawed. ②Allowing human organs to be sold will inevitably lead to a situation in which only the rich will be able to afford transplants. This is so because ③ whenever something scarce is bought and sold as a commodity, the price always goes up. ④The law of supply and demand requires it.

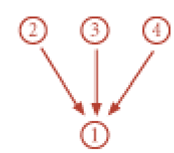
vertical pattern

horizontal pattern

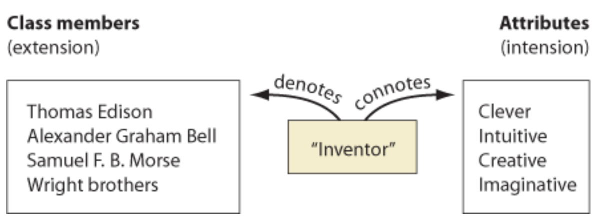
A distinct pattern of argumentation in some extended arguments. The *horizontal pattern* consists of a single argument in which two or more premises provide independent support for a single conclusion. If one of the premises were omitted, the other(s) would continue to support the conclusion in the same way. For example:

①The selling of human organs, such as hearts, kidneys, and corneas, should be outlawed. ②If this practice is allowed to get a foothold, people in desperate financial straits will start selling their own organs to pay their bills. Alternately, ③those with a criminal bent will take to killing healthy young people and selling their organs on the black market. ④In the final analysis, the buying and selling of human organs comes just too close to the buying and selling of life itself.

horizontal pattern



1.7 Language and Meaning

types of meaning	Linguistic expressions can have different kinds of meaning: • *Emotive meaning*: Expresses or evokes feelings • *Cognitive meaning:* Conveys information
emotive meaning	Statements having emotive meaning often make value claims. When such statements occur in arguments, the value claims should be disengaged from the emotive terminology and expressed as separate premises.
value claim	A claim that something is good, bad, right, wrong, better, worse, more important, or less important than some other thing. Such value claims are often the most important part of the cognitive meaning of emotive statements.
cognitive meaning	Cognitive meanings can be defective in two ways: • *Vagueness*: The meaning is blurred. • *Ambiguity*: More than one clearly distinct meaning is possible. Ambiguity and vagueness are important in logic because there are countless occasions in which the evaluation of an argument leads to the observation, "Well, that depends on what you mean by …" *If phraseology in an argument is vague or ambiguous, its meaning must be clarified before any evaluation can proceed.*
vague expression	An expression that allows for borderline cases in which it is impossible to tell if the expression applies or does not apply. Vague expressions often allow for a continuous range of interpretations. The meaning is hazy, obscure, and imprecise. How fresh does something have to be in order to be called "fresh"?
ambiguous expression	An expression that can be interpreted as having more than one clearly distinct meaning in a given context. For example, if one were to describe a beer as a *light* pilsner, does this mean that the beer is light in color, light in calories, or light in taste?
term	A word or phrase that can serve as the subject of a statement. Terms include: • Proper names (Napoleon, North Dakota, etc.) • Common names (animal, house, etc.) • Descriptive phrases (author of *Hamlet*, books in my library, etc.)
intension (connation)	Intensional meaning of a term refers to the attributes that the term connotes. For example, the intensional meaning of the term "cat" consists of the attributes of being furry, of having four legs, of moving in a certain way, of emitting certain sounds, etc.
conventional connotation	The *conventional connotation* of a term includes the attributes that the term *commonly* calls forth in the minds of competent speakers of the language. The connotation of a term remains more or less the same from person to person and from time to time.
extension (denotation)	Extensional meaning of a term refers to the members of the class that the term denotes. The extensional meaning of the term "cat" consists of cats themselves—all the cats in the universe.  ▶Hurley 2.2
empty extension	Terms with *empty extension* denote the empty (or "null") class, the class that has no members. For example, "unicorn," "leprechaun," "gnome," "elf," and "griffin."
mention (of a word)	"'Wherever' is an eight-letter word." In this statement, it is not the word itself that is the subject but rather the *quoted* word. In this statement, "wherever" is *mentioned*.
use (of a word)	"I will follow you *wherever* you go." In this statement, "wherever" is *used*.

1.7.1 Cognitive Language Defects: *Vagueness* and *Ambiguity*

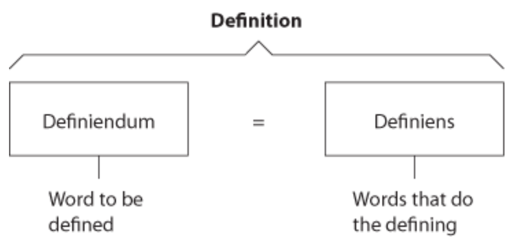
	Vagueness	Ambiguity
Definition	An vague expression **allows for borderline cases** in which it is impossible to tell if the expression applies or does not apply...A blur of meaning	An ambiguous expression **can be interpreted as having more than one clearly distinct meaning** in a given context...Uncertainty about the intended meaning, equivocation
Manifestation	Vague expressions **often allow for a continuous range of interpretations**. The meaning is hazy, obscure, and imprecise. Trouble arises only when the **language is not sufficiently precise for what the situation demands**.	Ambiguous terminology **allows for multiple discrete interpretations**. Trouble arises from mixing up otherwise clear meanings; when **more than one interpretation is plausible**.
Typical Words	“love,” “happiness,” “peace,” “excessive,” “fresh,” “rich,” “poor,” “normal,” “conservative,” and “polluted”	“light,” “proper,” “critical,” “stress,” “mad,” “inflate,” “chest,” “bank,” “sound,” and “race”
Key Question	Can I tell **with any precision** whether this word or statement applies to a given situation? For example: How fresh does something have to be in order to be called “fresh”?	Is it obvious **which interpretation or meaning is correct** in this context? For example: Professor Nobody saw the student in the corner of the lecture hall with binoculars.
Clarification Strategy	Use more precision in definition or description.	Use more clear-cut, definite, definitive, express, specific, unambiguous, unequivocal terminology.
Contextual Assessment	**Many forms of expression are ambiguous in one context and vague in another.** For example, the word “slow” in one context could mean either mentally challenged or physically slow, but when the word refers to physical slowness, it could be vague. How slow is slow?	

1.8 Definitions

definition

A word or group of words that assigns a meaning to a word or group of words:

- *Definiendum*: The word or group of words being defined
- *Definiens*: The word or group of words that does the defining



Definitions can serve different purposes, so there are different kinds of definitions

Stipulative definitions

Assign a meaning to a word when it first comes into use. This may involve either coining a new word or giving a new meaning to an old word. The purpose of a stipulative definition is usually to replace a more complex expression with a simpler one. Because a stipulative definition is a completely arbitrary assignment of a meaning to a word for the first time, there can be no such thing as a "true" or "false" stipulative definition.

Lexical definitions

Report the meaning a word has within a community of users. Dictionary definitions are all instances of lexical definitions. In contrast with a stipulative definition, which assigns a meaning to a word for the first time, a lexical definition may be true or false depending on whether it does or does not report the way a word is actually used. Also, lexical definitions are useful for eliminating ambiguity.

Precising definitions

Reduce the vagueness of a word. The definition "'Poor' means having an annual income of less than $10,000 and a net worth of less than $20,000" is an example of a precising definition. Whenever words are taken from ordinary usage and used in a highly systematic context such as science, mathematics, medicine, or law, they must always be clarified by means of a precising definition.

Theoretical definitions

Appeal to a theory to characterize whatever the term denotes. Such a definition provides a way of viewing or conceiving these entities that suggests deductive consequences, further investigation (experimental or otherwise), and whatever else would be entailed by the acceptance of a theory governing these entities. The definition of the term "heat" found in texts dealing with the kinetic theory of heat is a good example: "'Heat' means the energy associated with the random motion of the molecules of a substance." Many terms in philosophy, such as "substance," "form," "cause," "change," "idea," "good," "mind," and "God," have been given theoretical definitions.

Persuasive definitions

Influence the attitudes of the community of users regarding whatever the word denotes. Persuasive definitions amount to a certain synthesis of stipulative, lexical, and, possibly, theoretical definitions backed by the rhetorical motive to engender a certain attitude. As a result of this synthesis, a persuasive definition masquerades as an honest assignment of meaning to a term while condemning or blessing with approval the subject matter of the *definiendum*. For example:

- "Abortion" means the ruthless murdering of innocent children.
- "Abortion" means a safe and established surgical procedure whereby a woman is relieved of an unwanted burden.

1.8.1 Rules for Lexical Definitions

In reporting the meaning a word has within a community of users, a good lexical definition should:

- Conform to the standards of proper grammar.
- Convey the essential meaning of the word being defined.
- Be neither too broad nor too narrow.
- Avoid circularity.
- Not be negative when it can be affirmative.
- Avoid figurative, obscure, vague, or ambiguous language.
- Avoid affective terminology.
- Indicate the context to which the *definiens* pertains.

1.8.2 Definitional Techniques 1: *Extensional definitions*

extensional (denotative**) definition**	A definition that assigns a meaning to a term by indicating the members of the class that the *definiendum* denotes. There are at least three ways of indicating the members of a class: pointing to them, naming them individually, and naming them in groups. The three kinds of definitions that result: • *Demonstrative definitions* "point" to these things. • *Enumerative definitions* name individuals that the word denotes. • *Definitions by subclass* identify subclasses of these things.
demonstrative (ostensive) definition	Probably the most primitive form of definition. All one need know to understand such a definition is the meaning of pointing. Such definitions may be either partial or complete, depending on whether all or only some of the members of the class denoted by the *definiendum* are pointed to. For example, someone points to all the chairs in the room to indicate the meaning of "chair."
enumerative definition	Assigns a meaning to a term by naming the members of the class the term denotes. Like demonstrative definitions, they may also be either partial or complete. For example: "Actress" means a person such as Nicole Kidman, Emma Thompson, or Natalie Portman. "Baltic state" means Estonia, Latvia, or Lithuania.
definition by subclass	Assigns a meaning to a term by naming subclasses of the class denoted by the term. Such a definition, too, may be either partial or complete, depending on whether the subclasses named, when taken together, include all the members of the class or only some of them. For example: "Tree" means an oak, pine, elm, spruce, maple, and the like.
intension determines extension	The principle that *intension determines extension* underlies the fact that all extensional definitions suffer serious deficiencies. For example, in the case of the *demonstrative definition* of the word "chair," if all the chairs pointed to are made of wood, observers might get the idea that "chair" means "wood" instead of something to sit on.

1.8.3 Definitional Techniques 2: *Intensional definitions*

intensional (connotative**) definition**	A definition assigns a meaning to a word by indicating the qualities or attributes that the word connotes. Because at least four strategies may be used to indicate the attributes a word connotes, there are at least four kinds of intensional definitions: • *Synonymous definitions* use synonyms. • *Etymological definitions* disclose the word's ancestry. • *Operational definitions* specify experimental procedures. • *Definitions by genus and difference* identify a difference within a genus (set).
synonymous definition	A definition in which the *definiens* is a single word that connotes the same attributes as the *definiendum*. In other words, the *definiens* is a synonym of the word being defined. For example: "Physician" means doctor.
etymological definition	Assigns a meaning to a word by disclosing the word's ancestry in both its own language and other languages. Most ordinary English words have linguistic ancestors in Old or Middle English, Greek, Latin, etc. For example, the English word "captain" derives from the Latin noun *caput*, which means head.
operational definition	Assigns a meaning to a word by specifying certain experimental procedures that determine whether or not the word applies to a certain thing. For example: One substance is "harder than" another if and only if one scratches the other when the two are rubbed together.
definition by genus and difference	A assigns a meaning to a term by identifying a genus term and one or more difference words that, when combined, convey the meaning of the term being defined. *Definition by genus and difference is more generally applicable and achieves more adequate results than any of the other kinds of intensional definition.*
procedure for constructing definitions by genus and difference	1. Select a term that is more general than the term to be defined (the genus). 2. Identify a difference or subset within the genus that specifies the meaning of the term being defined. For example:

Species		Difference	Genus
"Daughter"	*means*	female	offspring
"Husband"	*means*	married	man
"Doe"	*means*	female	deer
"Fawn"	*means*	very young	deer
"Skyscraper"	*means*	very tall	building

1.8.4 Correlating Definitional Techniques with Types of Definitions

■ Extensional
■ Intensional

Technique	Stipulative	Lexical	Precising	Theoretical	Persuasive
	Produce This Type of Definition				
Demonstrative	yes	yes	no	(unusual)	(unusual)
Enumerative	yes	yes	no	(unusual)	(unusual)
Subclass	yes	yes	no	(unusual)	(unusual)
Synonymous	no	yes	no	no	no
Etymological	yes	yes	no	no	no
Operational	(limited)	yes	yes	(unusual)	(unusual)
Genus and Difference	**yes**	**yes**	**yes**	**yes**	**yes**

1.9 Fallacies

fallacy	A defect in an argument that arises from a mistake in reasoning or the creation of an illusion that makes a bad argument appear good. The term *non sequitur* ("it does not follow") is another name for fallacy. Both deductive and inductive arguments may contain fallacies; if they do, they are either unsound or uncogent, depending on the kind of argument. Fallacies are usually divided into two groups: • ***Formal***: Detectable by analyzing the form of an argument. Fallacies of this kind are found only in deductive arguments that have identifiable forms. For example, see sections 2.9 and 3.5 below. • ***Informal***: Detectable only by analyzing the content of an argument. **Fallacies that occur in real-life argumentation may be hard to detect:** • They may not exactly fit the pattern of the named fallacies. • They may involve two or more fallacies woven together in a single passage. **Three factors underlie the commission of fallacies in real-life argumentation:** • The intent of the arguer (the arguer may intend to mislead someone). • Mental carelessness combined with unchecked emotions. • Unexamined presuppositions in the arguer's worldview.
Informal fallacies	A defect in an argument that can be detected only by examining the content of the argument. Consider the following example: My house is made of atoms. Atoms are invisible. Therefore, my house is invisible. To detect this fallacy one must know something about houses—namely, that they are large visible objects, and even though their atomic components are invisible, this does not mean that the houses themselves are invisible. Since the time of Aristotle, logicians have attempted to classify the various informal fallacies. Here is a useful way to classify common informal fallacies:

Type	Typical Characteristic
Fallacies of Relevance	The premises are not relevant to the conclusion.
Fallacies of Weak Induction:	The premises may be relevant to the conclusion, but they supply insufficient support for the conclusion.
Fallacies of Presumption:	The premises presume what they purport to prove.
Fallacies of Ambiguity:	The conclusion depends on some kind of linguistic ambiguity.
Fallacies of Illicit Transference:	An attribute is incorrectly transferred from the parts of something onto the whole or from the whole onto the parts.

The common informal fallacies listed below involve errors that occur so often that they have been given specific names.

1.9.1 Common Informal Fallacies

Fallacies of Relevance	**▼ The premises are not relevant to the conclusion**
Accident	A general rule is applied to a specific case it was not intended to cover.
Appeal to force	Arguer threatens the reader/listener.
Appeal to pity	Arguer elicits pity from the reader/listener.
Appeal to the people	Arguer incites a mob mentality (direct form) or appeals to our desire for security, love, or respect (indirect form). This fallacy includes appeal to fear, the bandwagon argument, appeal to vanity, appeal to snobbery, and appeal to tradition.
Argument against the person	Arguer personally attacks an opposing arguer by: • verbally abusing the opponent (*ad hominem* abusive), • presenting the opponent as predisposed to argue as he or she does (*ad hominen* circumstantial), or • presenting the opponent as a hypocrite (*tu quoque*). Note: For this fallacy to occur, there must be two arguers.
Missing the point	Arguer draws a conclusion different from the one supported by the premises. Note: Do not cite this fallacy if another fallacy fits.
Red herring	Arguer leads the reader/listener off the track (with a misleading scent).
Straw man	Arguer distorts an opponent's argument and then attacks the distorted argument. Note: For this fallacy to occur, there must be two arguers.
Fallacies of Weak Induction	**▼ The premises offer insufficient support for the conclusion**
Appeal to ignorance	Premises report that nothing is known or proved about some subject, and then a conclusion is drawn about that subject.
Appeal to unqualified authority	Arguer cites an untrustworthy authority.
False cause	Conclusion depends on a nonexistent or minor causal connection. This fallacy has four forms: • *post hoc ergo propter hoc* • *non causa pro causa* • oversimplified cause • gambler's fallacy.
Hasty generalization	A general conclusion is drawn from an atypical sample.
Slippery slope	Conclusion depends on an unlikely chain reaction of causes.
Weak analogy	Conclusion depends on a defective analogy (similarity).
Fallacies of Presumption	**▼ The premises presume what they purport to prove**
Begging the question:	Arguer creates the illusion that inadequate premises are adequate by leaving out a key premise, restating the conclusion as a premise, or reasoning in a circle.
Complex question	Multiple questions are concealed in a single question.
False dichotomy	An "either … or …" (disjunctive) premise hides additional alternatives.
Suppressed evidence	Arguer ignores important evidence that requires a different conclusion.
Fallacies of Ambiguity	**▼ The conclusion depends on some kind of linguistic ambiguity**
Equivocation	Conclusion depends on a shift in meaning of a word or phrase.
Amphiboly	Conclusion depends on an incorrect interpretation of an ambiguous statement made by someone other than the arguer.
Fallacies of Illicit Transference	**▼ Incorrect attribution from parts to whole or from whole to parts**
Composition	An attribute is incorrectly transferred from the parts to the whole.
Division	An attribute is incorrectly transferred from the whole to the parts.

2. Categorical Logic

2.1 The Basics of Categorical Propositions

categorical proposition

A proposition that relates two classes, or categories. The classes are denoted respectively by the subject (S) term and the predicate (P) term. The proposition asserts that either all or part of the class denoted by S is included in or excluded from the class denoted by P.

standard form

The standard-form of a categorical proposition is:

<quantifier> **<S>** <copula> **<P>**

Eg: All S are P

A, E, I, and O propositions

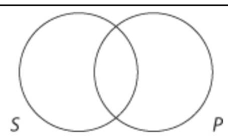
The names of the four types of categorical propositions:

A	All S are P.
E	No S are P.
I	Some S are P.
O	Some S are not P.

distribution

Universal (A and E) statements distribute their subject terms. Negative (E and O) statements distribute their predicate terms. Briefly: Universals distribute Subjects, and Negatives distribute Predicates. So, A statements distribute the Subject, E statements distribute both terms, I statements distribute neither term, and O statements distribute the Predicate.

Venn diagram



An arrangement of overlapping circles in which each circle represents the class denoted by a term in a categorical proposition. Every categorical proposition has exactly two terms (S and P), so the Venn diagram for a single categorical proposition consists of two overlapping circles.

Each circle is labeled so that it represents one of the terms in the proposition. In general, the left-hand circle represents the subject (S) term, and the right-hand circle the predicate (P) term.

- If an area is ***shaded***, there are ***no items*** in it.
- If an area contains an "***x***" there is ***at least one item*** in it.

Aristotelian standpoint

From the Aristotelian standpoint the first two *universal* propositions (A and E) can convey evidence about existence. When things exist, the Aristotelian standpoint recognizes their existence, and universal statements about those things have existential import.

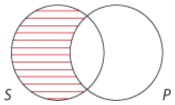
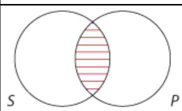
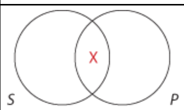
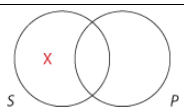
Boolean standpoint

From the Boolean standpoint, the first two *universal* propositions (A and E) imply nothing about the existence of the things denoted by S. When things exist, the Boolean standpoint does not recognize their existence, and universal statements about those things have no existential import.

interpreting universal propositions

The Aristotelian and Boolean standpoints are alternative sets of "ground rules" for interpreting the meaning of universal propositions. Either standpoint can be adopted for any categorical proposition or any argument composed of categorical propositions.

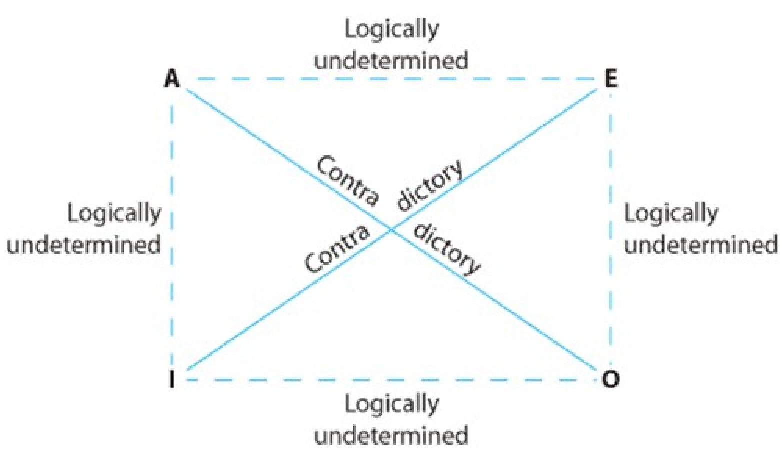
2.2 Categorical Propositions with Venn Diagrams

	Categorical Proposition	**Venn Diagram** Shading = emptiness | X = existence	**Boolean** Standpoint	**Aristotelian** Standpoint
A	**All S are P** ***(universal)***	 **No members of S are outside P.**	Existence not implied.	Existence implied if referring to actually existing things (real beings).
E	**No S are P** ***(universal)***	 **No members of S are inside P.**		
I	**Some S are P** ***(particular)***	 **At least one S exists that is a P.**	The word "some" implies existence. Eg: "Some mammals are zebras" asserts that at least one mammal exists that is a zebra.	
O	**Some S are not P** ***(particular)***	 **At least one S exists that is not a P.**	The word "some" implies existence. Eg: "Some mammals are not zebras" asserts that at least one mammal exists that is not a zebra.	

2.3 Quantity, Quality, and Distribution

Proposition	Letter Name	Quantity	Quality	Terms Distributed
All S are P	A	universal	affirmative	S
No S are P	E	universal	negative	S and P
Some S are P	I	particular	affirmative	none
Some S are not P	O	particular	negative	P

2.4 The Modern Square of Opposition



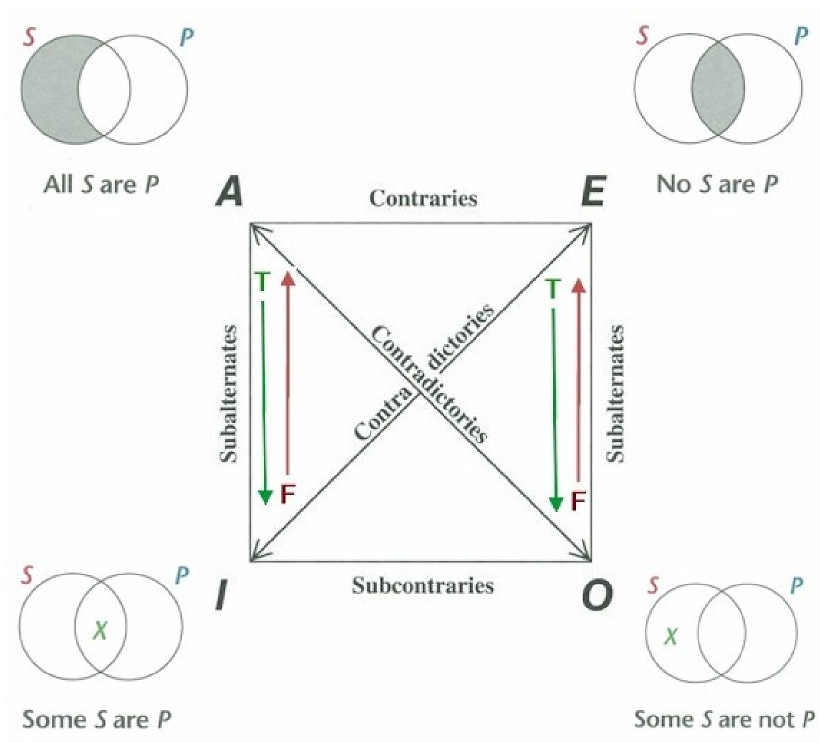
Contradictory relation

If two propositions are related by the *contradictory* relation, they necessarily have opposite truth value. Thus, if a certain A proposition is given as true, the corresponding O proposition must be false. Similarly, if a certain I proposition is given as false, the corresponding E proposition must be true.

Logically undetermined relations

Given the truth value of an A or O proposition, nothing can be determined about the truth value of the corresponding E or I propositions. And, given the truth value of an E or I proposition, nothing can be determined about the truth value of the corresponding A or O propositions. These propositions have *logically undetermined truth value*. Like all propositions, they do have a truth value, *but logic alone cannot determine what it is.*

2.5 Traditional Square of Opposition



Contradictory	**Opposite truth value** (the same as in the Modern Square of Opposition)
Contrary	**At least one is false** (not both true)
Subcontrary	**At least one is true** (not both false)
Subalternation	• **Truth flows downward** • **Falsity flows upward**

Shading = emptiness | **X** = existence

2.6 Existential Fallacy

existential fallacy	A formal fallacy that occurs if an argument is invalid *merely because the premise lacks existential* import. Such arguments always have a universal premise and a particular conclusion. However, not every inference having a universal premise and a particular conclusion commits the existential fallacy. From both the **Aristotelian** and the **Boolean standpoints**, universal propositions about things that do not exist (are not real), lead to the existential fallacy.

Existential Fallacy Examples	Aristotelian Standpoint	Boolean Standpoint
All zebras are mammals. ----------------------------------- Therefore, some zebras are mammals.	**Valid:** Refers to actually existing things.	**Invalid:** Existential fallacy
All zombies are mammals. ----------------------------------- Therefore, some zombies are mammals.	**Invalid:** Existential fallacy	

2.7 Logically Equivalent Statement Forms

Conversion, obversion, and contraposition are operations that can be performed on a categorical proposition, resulting in a new statement that may or may not have the same meaning and truth value as the original statement.

2.7.1 Conversion

Switch Subject and Predicate terms:

Given Statement	Converse	Truth Value
E: No A are B	**No B are A**	Same truth value as given statement
I: Some A are B	**Some B are A**	
A: All A are B	**All B are A**	Undetermined truth value
O: Some A are not B	**Some B are not A**	

2.7.2 Obversion

Change Quality; replace Predicate with *term compliment*

Given Statement	Obverse	Truth Value
A: All A are B	**No A are non-B**	Same truth value as given statement
E: No A are B	**All A are non-B**	
I: Some A are B	**Some A are not non-B**	
O: Some A are not B	**Some A are non-B**	

2.7.3 Contraposition

Switch Subject and Predicate terms; replace each with its *term complement*:

Given Statement	Converse	Truth Value
A: All A are B	**All non-B are non-A**	Same truth value as given statement
O: Some A are not B	**Some non-B are not non-A**	
E: No A are B	**No non-B are non-A**	Undetermined truth value
I: Some A are B	**Some non-B are non-A**	

2.8 Categorical Syllogisms

categorical syllogism	A deductive argument that has three categorical propositions and is able to be translated into ***standard syllogistic form***.
class complement **term complement**	The complement of a class is the group consisting of everything outside the class. For example, the complement of the class of dogs is the group that includes everything that is not a dog (cats, fish, trees, and so on). The ***term complement*** is the word or group of words that denotes the ***class complement***. For terms consisting of a single word, the term complement is usually formed by simply attaching the prefix "non" to the term. Thus, the complement of the term "dog" is "non-dog," the complement of the term "book" is "non-book," and so on.
form of a syllogism	The ***form*** of a syllogism is determined by its ***mood*** and ***figure***. After a categorical syllogism has been put into ***standard form***, its validity or invalidity can be determined by merely inspecting its form.
mood	Denoted by the letter names (A, E, I, O), of its constituent propositions. The letter for the major premise is listed first, then the letter for the minor premise, and finally the letter for the conclusion.
figure	Determined by the position of the occurrences of the middle term in the premises. There are four figures.

2.8.1 Standard Form: *Categorical Syllogism*

Standard Syllogistic Form	
1. Quantifier ____________ copula ____________	**Major premise** - contains **major term**
2. Quantifier ____________ copula ____________	**Minor premise** - contains **minor term**
3. Quantifier (minor term) **copula** (major term)	**Conclusion**

2.8.2 Symbolizing Syllogistic Forms

To symbolize the four figures:

1. Drop the quantifiers and copulas.
2. Determine the positions of the three terms in the syllogism:

   S =the Subject of the conclusion (minor term)
   P = the Predicate of the conclusion (major term)
   M = the middle term (occurs once in each premise but not in the conclusion)

Figure 1	Figure 2	Figure 3	Figure 4
M P S M S P	P M S M S P	M P M S S P	P M M S S P

For example, the ***form*** of the following syllogism is **EIO-4** (***mood* EIO + *figure* 4**):

E: No cyborgs are **Earthlings**. — **major term:** cyborgs
I: Some **Earthlings** are Martians. — **minor term:** Martians
O: Therefore, some Martians are not cyborgs. — **Conclusion**

2.8.3 Unconditionally Valid Syllogistic Forms (figures + moods)

Figure 1	Figure 2	Figure 3	Figure 4
M P S M S P	P M S M S P	M P M S S P	P M M S S P
AAA EAE AII EIO	EAE AEE EIO AOO	IAI AII OAO EIO	AEE IAI EIO

2.8.4 Conditionally Valid Syllogistic Forms

Figure 1	Figure 2	Figure 3	Figure 4	
M P S M S P	P M S M S P	M P M S S P	P M M S S P	**Required Condition**
AAI EAO	AEO EAO		AEO	S exists
		AAI EAO	EAO	M exists
			AAI	P exists

2.9 Rules for Categorical Syllogisms

Rule 1: **The middle term must be distributed at least once.**
Fallacy: Undistributed middle

Rule 2: **If a term is distributed in the conclusion, then it must be distributed in the premise.**
Fallacy: Illicit major; illicit minor

Rule 3: **Two negative premises are not allowed.**
Fallacy: Exclusive premises

Rule 4: **A negative premise requires a negative conclusion, and a negative conclusion requires a negative premise.**
Fallacy: Drawing an affirmative conclusion from a negative premise; drawing a negative conclusion from affirmative premises

Rule 5: **If both premises are universal, the conclusion cannot be particular.**
Fallacy: Existential fallacy

NOTE: If only Rule 5 is broken, the syllogism is valid from the Aristotelian standpoint if the critical term denotes actually existing things.

3. Propositional Logic

3.1 Hurley Symbol Set

Operation	Hurley Operator	Alternative Symbols (QWERTY keyboard)		Other Common Symbols
Negation (Not)	~ (tilde)	~	(tilde)	¬
Disjunction (Or)	ᵛ (wedge)	ᵛ	(lower-case "v")	∨
Conditional (If…Then)	⊃ (horseshoe)	>	(right-angle bracket)	→
Conjunction (And)	• (dot)	&	(ampersand)	∧
Biconditional (If and Only If)	≡ (triple bar)	=	(equal sign) Same as the identity operator.	↔
Existential Quantification	∃x	3x	(number three-lower-case "x")	∃x
Universal Quantification	(x)	(x)	(lower-case "x" in parentheses)	∀x

3.2 Basic Symbolization in Propositional Logic

English Connectives	Proposition Paraphrase | Notes	Operator	Statement Form
(Plus) ***And***	Sometimes 'and' is used to mean 'plus', as in 'Two and two are four.'	**none**	*p*
(Semicolon) ***;***	*p*; *q*. | In English, the semicolon functions as a sign of conjunction.	•	*p* • *q*
(Set Members) ***And***	Sometimes 'and' is used like this: 'Jack and Jill are a pair'.	**none**	*p*
<root>n't	Contractions like '*isn't*' and '*can't*' and '*wasn't*'	~	~*p*
A<root>	Terms like '*amoral'* and '*apolitical'* must be evaluated in context.	~ / **none**	~*p* / *p*
Alternatively	*p*, alternatively *q*.	∨	*p* ∨ *q*
Although	*p*, although *q*. / Although *p*, *q* | Emphasizes contrast between the conjuncts	•	*p* • *q*
And	*p* and *q* | Conjunction (p and q are conjuncts)	•	*p* • *q*
Believes that	S believes that *p*. | Simple statement—no connective	**none**	*p*
Both…and	Both *p* and *q*.	•	*p* • q
But	*p*, but *q*. | Emphasizes contrast between the conjuncts	•	*p* • *q*
Denied	That p is denied.	~	~*p*
Depends on	*p* depends on *q*.	⊃	*p* ⊃*q*
Either…or	Either *p* or *q*.	∨	*p* ∨ *q*
Entails	*p* entails *q*.	⊃	*p* ⊃*q*
Exactly when	*p* exactly when *q*.	≡	*p* ≡*q*
False	That *p* is false. / It is false that *p*.	~	~*p*

English Connectives	Proposition Paraphrase | Notes	Operator	Statement Form
Given that…	*p* given that *q* | *q* is a statement of a sufficient condition (see below) for p	$\supset$	$q \supset p$
Guarantees	*p* guarantees *q*.	$\supset$	$p \supset q$
However	*p*, however, *q*. | Emphasizes contrast between the conjuncts	$\bullet$	$p \bullet q$
If	*q*, if *p*. / If *p*, *q*.	$\supset$	$p \supset q$
If, and only if	*p* if, and only if, *q* | Biconditional (p and q are conditions)	$\equiv$	$p \equiv q$
If…then	If *p*, then *q* | Conditional (*p* is the antecedent, *q* is the consequent)	$\supset$	$p \supset q$
Im<root>	Terms like '*impermanent'* and '*immobile'* usually express a negation.	$\sim$	$\sim p$
Implies	*p* implies *q*.	$\supset$	$p \supset q$
In spite of the fact that	*p*, in spite of the fact that *q*. | Emphasizes contrast between the conjuncts	$\bullet$	$p \bullet q$
In<root>	Terms like '*inadequate'* and '*ineffective'* usually express a negation.	$\sim$	$\sim p$
Is conditional upon	*p* is conditional upon *q*.	$\supset$	$p \supset q$
Is contingent upon	*p* is contingent upon *q*.	$\supset$	$p \supset q$
Is dependent upon	*p* is dependent upon *q*.	$\supset$	$p \supset q$
Just in case	*p* just in case *q*.	$\equiv$	$\mathrm{p} \equiv \mathrm{q}$
Leads to	*p* leads to *q*.	$\supset$	$p \supset q$
Necessary and sufficient	*p* is necessary and sufficient for *q*. | Each condition is necessary and sufficient for the other.	$\equiv$	$p \equiv q$
Necessary for	*q* is necessary for *p*.| A necessary condition statement is in the consequent position.	$\supset$	$p \supset q$
Neither…nor (1)	Neither *p* nor *q*. | Expressing the conjunction of two negations.	$\bullet$	$(\sim p \bullet \sim q)$
Neither…nor (2)	Neither *p* nor *q*. | Expressing the negation of a disjunction.	$\vee$	$\sim(p \vee q)$
Non<root>	Terms like '*noncommittal'* and '*nonsense'* usually express a negation.	$\sim$	$\sim p$
Not	It is not the case that *p*. | Negation (p is the scope)	$\sim$	$\sim p$
Not true	That p is not true. | It is not true that p.	$\sim$	$\sim p$
On condition that	On the condition that *p*, *q*.	$\supset$	$p \supset q$
Only if	*p only* if *q*. / Only if *q*, *p*. | 'If' usually precedes the antecedent—but not for 'only if'.	$\supset$	$p \supset q$
Or	*p* or *q* | Disjunction (p and q are disjuncts)	$\vee$	$p \vee q$

English Connectives	Proposition Paraphrase | Notes	Operator	Statement Form
Provided	*q* provided *p.*	$\supset$	$p \supset q$
Requires	*p* requires *q.*	$\supset$	$p \supset q$
Sufficient for	*p* is sufficient for *q* | A sufficient condition statement is in the antecedent position.	$\supset$	$p \supset q$
The one thing that leads to	*p* is the one thing that leads to *q.*	$\equiv$	$p \equiv q$
Un<root>	Terms like '*unhappy*' and '*unorganized*' must be evaluated in context.	~ / **none**	$\sim p$ / p
Unless (1)	*p* unless *q.* | 'Unless' can also be symbolized: $\sim q \supset p$.	$\vee$	$p \vee q$
Unless (2)	*p* unless *q.* | 'Unless' can also be symbolized: $p \vee q$.	$\supset$	$\sim q \supset p$
When, and only when	*p* when, and only when, *q*.	$\equiv$	$p \equiv q$
Whenever	*q*, whenever *p.*	$\supset$	$p \supset q$
At least one	At least one of the set *p and q* is the case. | '*and*' does not express conjunction	$\vee$	$p \vee q$
At least one…and at most one	At least one of the set *p and q* is the case, and at most one is the case.	•	$(p \vee q) \bullet \sim(p \bullet q)$

3.3 Truth Tables for Logical Operators (Connectives in Compound Statements)

Conjunction	Negation	Disjunction	Conditional	Biconditional
P • Q	~ P	P ∨ Q	P ⊃ Q	P ≡ Q
T *(T)* T	*(F)* T	T *(T)* T	T *(T)* T	T *(T)* T
T *(F)* F	*(T)* F	T *(T)* F	T *(F)* F	T *(F)* F
F *(F)* T		F *(T)* T	F *(T)* T	F *(F)* T
F *(F)* F		F *(F)* F	F *(T)* F	F *(T)* F

3.4 Valid Inference Forms

Modus Ponens	Modus Tollens
p ⊃ q	**p ⊃ q**
p	**~q**
q	**~p**

3.5 Invalid Inference Forms (Formal Fallacies)

Affirming the Consequent	Denying the Antecedent
p ⊃ q	**p ⊃ q**
q	**~p**
p	**~q**

3.6 Rules of Implication

Modus ponens (MP)
$p \supset q$
p
q

Hypothetical syllogism (HS)
$p \supset q$
$q \supset r$
$p \supset r$

Constructive dilemma (CD)
$(p \supset q) \bullet (r \supset s)$
$p \vee r$
$q \vee s$

Conjunction (Conj)
p
q
$p \bullet q$

Modus tollens (MT)
$p \supset q$
$\sim q$
$\sim p$

Disjunctive syllogism (DS)
$p \vee q$
$\sim p$
q

Simplification (Simp)
$p \bullet q$
p

Addition (Add)
p
$p \vee q$

3.7 Rules of Replacement

De Morgan's rule (DM)	$\sim(p \bullet q) :: (\sim p \vee \sim q)$
	$\sim(p \vee q) :: (\sim p \bullet \sim q)$
Commutativity (Com)	$(p \vee q) :: (q \vee p)$
	$(p \bullet q) :: (q \bullet p)$
Associativity (Assoc)	$[p \vee (q \vee r)] :: [(p \vee q) \vee r]$
	$[p \bullet (q \bullet r)] :: [(p \bullet q) \bullet r]$
Distribution (Dist)	$[p \bullet (q \vee r)] :: [(p \bullet q) \vee (p \bullet r)]$
	$[p \vee (q \bullet r)] :: [(p \vee q) \bullet (p \vee r)]$
Double negation (DN)	$p :: \sim\sim p$
Transposition (Trans)	$(p \supset q) :: (\sim q \supset \sim p)$
Material implication (Impl)	$(p \supset q) :: (\sim p \vee q)$
Material equivalence (Equiv)	$(p \equiv q) :: [(p \supset q) \bullet (q \supset p)]$
	$(p \equiv q) :: [(p \bullet q) \vee (\sim p \bullet \sim q)]$
Exportation (Exp)	$[(p \bullet q) \supset r] :: [p \supset (q \supset r)]$
Tautology (Taut)	$p :: (p \vee p)$
	$p :: (p \bullet p)$

3.8 Conditional Proof (CP)

1. ***Assume*** the antecedent of a required conditional statement (ACP) on the first line of an indented sequence.
2. Derive the consequent on a subsequent line.
3. Discharge the indented sequence in a conditional statement that is exactly the one to be obtained.

—		
—		
—		/ __
	p	ACP
	—	
	—	
	—	
	q	
$p \supset q$		CP

3.9 Indirect Proof (IP)

1. ***Assume*** the negation of a required statement (AIP) on the first line of an indented sequence.
2. Derive a contradiction on a subsequent line.
3. Discharge the indented sequence by asserting the negation of the assumed statement.

—		
—		
—		/__
	p	AIP
	—	
	—	
	—	
	$q \bullet \sim q$	
$\sim p$		IP

4. Predicate Logic

4.1 Subjects and Predicates

➲	
𝒮	In categorical logic, the ***S*** term is the **subject term.**
𝒫	In categorical logic, the ***P*** term is the **predicate term**.
predicates	In predicate logic, **both subject and predicate terms are predicates.** That is, each denotes a set of objects (eg: the set of humans is completely included in the set of mammals).

4.2 Translating the Basic Statement Forms

A	All 𝒮 are 𝒫.	**(x)(𝒮x⊃ 𝒫x)**
E	No 𝒮 are 𝒫.	**(x)(𝒮x ⊃ ~ 𝒫x)**
I	Some 𝒮 are 𝒫.	**(∃x)(𝒮x • 𝒫x)**
O	Some 𝒮 are not 𝒫.	**(∃x)(𝒮x • ~ 𝒫x)**

Statement Form		English Example	Translation	Logical Meaning
A	All 𝒮 are 𝒫.	All surgeons are doctors	**(x)(Sx ⊃ Dx)**	For all / any x, if x is an *S*, then x is a *D*
E	No 𝒮 are 𝒫.	No crabs are crows.	**(x)(Cx ⊃ ~Rx)**	For all / any x, if x is an *C*, then x is not an *R*
I	Some 𝒮 are 𝒫.	Some apples are pippins.	**∃x(Ax • Px)**	There is at least one x such that x is an *A* and x is a *P*.
O	Some 𝒮 are not 𝒫.	Some chess pieces are not art.	**∃x(Cx • ~Ax)**	There is at least one x such that x is a *C* and x is not an *A*.

4.3 Symbolization in Predicate Logic

4.3.1 General Categorical Statements Using the Universal Quantifier (x)

General Categorical Statements Using (x)	Symbolization	Notes
Everything is an A.	**(x)Ax**	**~(∃x)~Ax (Q)**
Not everything is an A.	**~(x)Ax**	**(∃x)~Ax (Q)**
Everything is a non-A.	**(x)~Ax**	**~(∃x)Ax (Q)**
Not everything is a non-A.	**~(x)~Ax**	**(∃x)Ax (Q)**
All As are Bs.	**(x)(Ax⊃ Bx)**	
No As are Bs.	**(x)(Ax ⊃ ~Bx)**	
Everything is an-A and a B.	**(x)(Ax • Bx)**	**(x)Ax • (x)Bx**
Only As are Bs.	**(x)(Bx ⊃ Ax)**	Necessary condition

4.3.2 Typical English Statements Using the Universal Quantifier (x)

Typical English Statements Using (x)	Symbolization	Notes
1. Everything is movable.	**(x)Mx**	
2. Not everything is movable.	**~(x)Mx**	Negated quantifier
3. Nothing is movable.	**(x)~Mx**	
4. Everything is immovable.	**(x)~Mx**	3 and 4 are equivalent.
5. It's not true that everything is immovable.	**~(x)~Mx**	Negated quantifier
6. Sugar tastes sweet.	**(x)(Sx⊃ Tx)**	
7. If something is a piece of sugar, then it tastes sweet.	**(x)(Sx⊃ Tx)**	6 and 7 are equivalent.
8. Everything is either sweet or bitter.	**(x)(Sx ∨ Bx)**	
9. Either everything is sweet, or else everything is bitter.	**(x)Sx ∨ (x) Bx**	8 and 9 are not equivalent
10. Each person fears death.	**(x)(Px⊃ Fx)**	
11. Everyone fears death.	**(x)(Px⊃ Fx)**	10 and 11 are equivalent.
12. No one fears death.	**(x)(Px⊃ ~Fx)**	
13. Not everyone fears death.	**~(x)(Px⊃ Fx)**	Negated quantifier
14. All honest people fear death.	**(x)(Px • Hx) ⊃ Fx**	
15. Everyone who is honest fears death.	**(x)(Px • Hx) ⊃ Fx**	14 and 15 are equivalent.
16. Not all honest people fear death.	**~(x)(Px • Hx) ⊃ Fx**	Negated quantifier
17. Anyone who doesn't fear death isn't honest.	**(x)(Px • ~Fx) ⊃ ~Hx**	
18. Although all honest people fear death, Shirley doesn't.	**(x)[(Px • Hx) ⊃ Fx] • ~Fs**	
19. Everyone either is honest or fears death.	**(x)[Px ⊃ (Hx ∨ Fx)]**	
20. It is not the case that no dishonest people fear death.	**~(x)(Px • ~Hx) ⊃ ~Fx**	Negated quantifier
21. Either all human beings are mortal or none are.	**(x)(Hx⊃ Mx) ∨(x)(Hx⊃ ~Mx)**	
22. If all human beings are mortal, then not to fear death indicates not being human.	**(x)(Hx ⊃ Mx) ⊃(x)(~Fx ⊃ ~Hx)**	

4.3.3 General Categorical Statements Using the Existential Quantifier ∃x

General Categorical Statements Using ∃x	Symbolization	Notes
There is at least one A / There are some A.	**(∃x)Ax**	**~(x)~Ax (Q)**
There is nothing that is an A.	**~(∃x)Ax**	**(x)~Ax (Q)**
There is at least one non-A.	**(∃x)~Ax**	**~(x)Ax (Q)**
There is nothing that is a non-A.	**~(∃x)~Ax**	**(x)Ax (Q)**
Some As are Bs.	**(∃x)(Ax • Bx)**	
Some As are not Bs.	**(∃x)(Ax • ~Bx)**	
There is at least one x that is an A or a B.	**(∃x)(Ax ∨ Bx)**	**(∃x)Ax ∨ (∃x)Bx (*Q*∨)**
If anyone is an A, then…	**(∃x)(Ax ⊃ …)**	

4.3.4 Typical English Statements Using the Existential Quantifier ∃x

Typical English Statements Using ∃x	Symbolization	Notes
1. Some people are honest.	**(∃x)(Px • Hx)**	
2. Some people are not honest.	**(∃x)(Px • ~Hx)**	
3. Some honest people are mistreated.	**(∃x)(Px • Hx) • Mx**	
4. Some people are liars and thieves.	**(∃x)[Px • (Lx • Tx)]**	
5. It's not true that some people are honest.	**~(∃x)(Px • Hx)**	Negated quantifier
6. Some people are neither honest nor truthful.	**(∃x)[Px • ~(Hx ∨ Tx)]**	**(∃x)[Px • (~Hx • ~Tx)]**
7. Some things are neither expensive nor worthwhile.	**(∃x)~(Ex ∨ Wx)**	**(∃x)(~Ex • ~Wx)**
8. Some people are liars and some are thieves.	**(∃x)(Px • Lx) • (∃x)(Px • Tx)**	Compare with 4 above.
9. Some thieving liars are caught, and some aren't.	**(∃x)[(Tx • Lx) • Cx] • (∃x)[(Tx • Lx) • ~Cx]**	

4.4 Change of Quantifier Rule (CQ)

~(∃x)Ax	There is nothing that is an A.	::	**(x)~Ax**	Everything is a non-A.
~(x)Ax	Not everything is an A.	::	**(∃x)~Ax**	There is at least one non-A
~(∃x)~Ax	There is nothing that is a non-A.	::	**(x)Ax**	Everything is an A.
~(x)~Ax	Not everything is a non-A.	::	**(∃x)Ax**	There is at least one A.

4.5 Removing and Introducing Quantifiers

Remove Quantifiers	Introduce Quantifiers
Universal Instantiation (UI) All friendly people are generous. Anna is friendly. ∴ Anna is generous.	**Universal Generalization (UG)** All psychiatrists are doctors. All doctors are college graduates. ∴ All psychiatrists are college graduates.
Existential Instantiation (EI) All lawyers are college graduates. Some lawyers are golfers. ∴ Some golfers are college graduates.	**Existential Generalization (EG)** All tenors are singers. Placido Domingo is a tenor. ∴ There is at least one singer…

($a, b, c, \ldots u, v, w$ are individual constants; x, y, z are individual variables)

1. Universal instantiation (UI) $\quad \dfrac{(x)\mathcal{F}x}{\mathcal{F}y} \qquad \dfrac{(x)\mathcal{F}x}{\mathcal{F}a}$

2. Universal generalization (UG) $\quad \dfrac{\mathcal{F}y}{(x)\mathcal{F}x}$ *not allowed:* $\dfrac{\mathcal{F}a}{(x)\mathcal{F}x}$

   Restrictions: *(conditional and indirect proof)* (1) UG must not be used within the scope of an indented sequence if the instantial variable y is free in the first line of that sequence.

   (overlapping quantifiers) (2) UG must not be used if the instantial variable y is free in any preceding line obtained by EI.

3. Existential instantiation (EI) $\quad \dfrac{(\exists x)\mathcal{F}x}{\mathcal{F}a}$ *not allowed:* $\dfrac{(\exists x)\mathcal{F}x}{\mathcal{F}y}$

   Restriction: The existential name a must be a new name that does not appear in any previous line (including the conclusion line).

4. Existential generalization (EG) $\quad \dfrac{\mathcal{F}a}{(\exists x)\mathcal{F}x} \qquad \dfrac{\mathcal{F}y}{(\exists x)\mathcal{F}x}$

5. Advanced Logic Reference

5.1 Online Logic Resources

Site Name | Description | URL
A Comprehensive List of Logic Symbols http://en.wikipedia.org/wiki/List_of_logic_symbols
A Course in Advanced Logic - Kit Fine / NYU http://www.nyu.edu/gsas/dept/philo/courses/advlogic/
A list of classic texts by Greco-Roman and Eastern Authors http://classics.mit.edu/Browse/index.html
Automated Theorem Prover Beta version for classical predicate logic. http://logik.phl.univie.ac.at/~chris/gateway/formular-uk-ableitung.html
Bibliography of Non-Standard Logics A brief overview of various logic systems http://www.earlham.edu/~peters/courses/logsys/nonstbib.htm
blogic: A WebLogic Textbook Interactive textbook for introductory logic courses. Topics include: (i) Boolean searching (ii) propositional logic with truth-tables (iii) the logic of frequencies and probabilities (iv) modal logic and counterfactuals (v) quantification. Textbook includes interactive exercises. http://www.merlot.org/merlot/viewMaterial.htm?id=79868
Christian Gottschall's Gateway to Logic A collection of on-line logic programs (two require Java), offering parse trees, alpha graphs, proof checking, truth tables, Polish notation, and much, much more. http://logik.phl.univie.ac.at/~chris/formular-uk.html
Classical Logic Shapiro, Stewart, "Classical Logic", *The Stanford Encyclopedia of Philosophy (Winter 2009 Edition)*, Edward N. Zalta (ed.) http://plato.stanford.edu/entries/logic-classical/
Formal Logic This is a featured book on Wikibooks. It is an online, undergraduate college level textbook covering first order predicate logic with identity but omitting metalogical proofs. http://en.wikibooks.org/wiki/Formal_Logic
Fuzzy Logic Toolbox™ http://www.mathworks.com/products/fuzzy-logic/
Fuzzy Logic Tutorial http://www.seattlerobotics.org/encoder/mar98/fuz/flindex.html

KHAN Academy Online videos for Brain Teasers http://www.khanacademy.org/search?page_search_query=logic	
Lewis Carroll Puzzles As a teacher of logic and a lover of nonsense, Carroll designed entertaining puzzles to train people in systematic reasoning. In these puzzles he strings together a list of implications, purposefully inane so that the reader is not influenced by any preconceived opinions. The job of the reader is to use all the listed implications to arrive at an inescapable conclusion. You will get the general idea after a few examples. http://www.math.hawaii.edu/~hile/math100/logice.htm	
Lewis Carroll's Logic Game As a tool for solving logical puzzles, the diagrams are of little value when the number of variables exceeds4. http://www.cut-the-knot.org/LewisCarroll/LCGame.shtml	
Logic for Programming and Artificial Reasoning - Conference Excerpts https://books.google.com/books?id=ANAgZTWWNfoC&pg=PA132&lpg=PA132&dq=quantification+in+Abelian+equations&source=bl&ots=NpYGMNsMnD&sig=gnGf26A-Nybo1l-NdK8LYQNsY8A&hl=en&sa=X&ei=MWGHVLyuD4H3oASvl4GgDw&ved=0CDIQ6AEwAg#v=onepage&q=quantification%20in%20Abelian	
Logic QuizMaster A useful tool for testing comprehension of sentential and predicate logic basics. http://logic.ua.edu/cgi-bin/quizmaster	
Logic System Interrelationships For modal logic systems http://home.utah.edu/~nahaj/logic/structures/index.html	
Logic Systems A reference resource for mostly modal logic systems. http://home.utah.edu/~nahaj/logic/structures/systems/index.html	
Mediaeval Logic and Philosophy For anyone interested in mediaeval logic and philosophy broadly construed, this site offers an extensive list of downloadable period documents. http://pvspade.com/Logic/noframes/download.html	
Proof Checker Checks proofs submitted by the user. It supports Lemmon's calculus only. (see E.J. Lemmon's book Beginning Logic.) http://logik.phl.univie.ac.at/~chris/gateway/formular-uk-beweis.html	

Proof Designer Proof designer is a tool intended to help students who are beginning to learn how to write proofs. While proving theorems certainly is a creative task, there are many steps that actually are schematic and mathematicians have internalized them. http://www.merlot.org/merlot/viewMaterial.htm?id=78631
Propositional Logic Calculator A good tool for truth table analysis. http://logik.phl.univie.ac.at/~chris/cgi-bin/cgi-form?key=00000b5d
Sample Sorites A brief introduction. http://www.cut-the-knot.org/LewisCarroll/soriteses.shtml
The Association for Symbolic Logic Central meeting page for members, this includes sections of the Bulletin, the Journal, and the Newsletter of the Association. http://www.aslonline.org/
The *Stanford Encyclopedia of Philosophy* Table of Contents page. For logic terms, scroll down to the Logic entries http://plato.stanford.edu/contents.html
Venn diagrams A brief introduction. http://www.cut-the-knot.org/LewisCarroll/dunham.shtml
What is Logic? A detailed taxonomy of all things logic; from philosophical logic to higher order set theory to Lambda-calculi. http://www.rbjones.com/rbjpub/logic/log025.htm
Zeno's Coffeehouse You know it, you love it. BS your way into a solution of the most (a)typical problems. http://www.valdosta.edu/~rbarnett/phi/zeno.html

5.2 Video Links

5.2.1 Basic Logic Videos	Notes
The philosophical method - logic and argument	
Introduction to Logic (Standford)	
Propositional Logic	
If-Then Statements and Converses	
The Converse, Contrapositive, and Inverse of an If-Then Statement	
Complete List of Math and Logic Videos from *MathInSocietyVideos*	
5.2.2 Truth Tables	
Logic & Arguments - Truth Tables	
Truth Tables for Compound Statements	
Truth Tables for Conditional Statements	
Truth Table for the Biconditional Statement	
Truth Tables: Showing Statements are Equivalent	
5.2.3 Fallacies & Illusions	
Logic & Arguments - Logical Fallacies (formal & informal fallacies)	Overview
Informal Fallacies	A short list
The Fallacy Project: Examples of fallacies from advertising, politics, and popular culture	Comic relief
Hypercubes and Plato's Cave	Observing 4 dimensions?
The illusion of time : past, present and future all exist together	Naturalism: Version 1
The Evolution of the Laws of Physics - Lee Smolin (SETI Talks)	Naturalism: Version 2
5.2.4 Basic Set Theory	
*Introduction to Set Theory	
*Introduction to Subsets	
*Set Operations and Venn Diagrams - Part 1 of 2	
*Set Operations and Venn Diagrams - Part 2 of 2	
*Solving Problems Using Venn Diagrams	

5.3 Advanced Topics

5.3.1 Identity in Predicate Logic

Identity (Id) Rules of Inference

1. Prem.
   $a = a$
2. $a = b :: b = a$
3. $\mathscr{F}a$
   $\underline{a = b}$
   $\mathscr{F}b$

5.3.2 The Theory of Descriptions

According to Bertrand Russell, saying "The golden mountain does not exist" is really just a misleading way of saying "It is not the case that there is exactly one thing that is a mountain and is golden." Thus analyzed, it becomes clear that the proposition does not refer to anything, but simply denies an existential claim.

For instance, with the definitions of Mx as "x is a mountain" and Gx as "x is golden," the proposition that"*the* golden mountain does not exist" becomes:

$$\sim[(\exists x)(\mathbf{M}x \;\&\; \mathbf{G}x) \;\&\; \forall y((\mathbf{M}y \;\&\; \mathbf{G}y) \rightarrow y=x)]$$

Equivalently, in English, it is not the case that there is some object such that (1) it is a mountain, (2) it is golden, and (3) all objects that are mountains and golden are identical to it.

Since it does not refer to any "golden mountain," it does not need a Meinongian object to provide it with meaning. In fact, taking the latter formulation to be the true *logical form* of the statement, Russell construes the original's reference to a non-existent golden mountain as a matter of grammatical illusion. One dispels the illusion by making the grammatical form match the true logical form, and this is done through logical analysis. The idea that language could cast illusions that needed to be dispelled, some form of linguistic analysis was to be a prominent theme in analytic philosophy, both in its ideal language and ordinary language camps, through roughly 1960.

http://www.iep.utm.edu/analytic/

5.3.3 The Algebra of Logic Tradition

- 1. Introduction
- 2. 1847—The Beginnings of the Modern Versions of the Algebra of Logic
- 3. 1854—Boole's Final Presentation of his Algebra of Logic
- 4. Jevons: An Algebra of Logic Based on Total Operations
- 5. Peirce: Basing the Algebra of Logic on Subsumption
- 6. De Morgan and Peirce: Relations and Quantifiers in the Algebra of Logic
- 7. Schröder's systematization of the algebra of logic
- 8. Huntington: Axiomatic Investigations of the Algebra of Logic
- 9. Stone: Models for the Algebra of Logic
- 10. Skolem: Quantifier Elimination and Decidability
- 11. Tarski and the Revival of Algebraic Logic

http://plato.stanford.edu/entries/algebra-logic-tradition/

5.3.4 Gödel's Incompleteness Theorems

Gödel's two incompleteness theorems are among the most important results in modern logic, and have deep implications for various issues. They concern the limits of provability in formal axiomatic theories. The first incompleteness theorem states that in any consistent formal system *F* within which a certain amount of arithmetic can be carried out, there are statements of the language of *F* which can neither be proved nor disproved in *F*. According to the second incompleteness theorem, such a formal system cannot prove that the system itself is consistent (assuming it is indeed consistent). These results have had a great impact on the philosophy of mathematics and logic. There have been attempts to apply the results also in other areas of philosophy such as the philosophy of mind, but these attempted applications are more controversial. The present entry surveys the two incompleteness theorems and various issues surrounding them.

http://plato.stanford.edu/entries/goedel-incompleteness/

Research on the consequences of this great theorem continues to this day. Anyone with Internet access using a search engine like Alta Vista can find several hundred articles of highly varying quality on Godel's Theorem. Among the best things to read, though, is *Godel's Proof* by Ernest Nagel and James R. Newman, published in 1958 and released in paperback by New York University Press in 1983.

http://www.scientificamerican.com/article/what-is-godels-theorem/

5.3.5 Modal Logics

Logic	Symbol	Expressions Symbolized
Modal Logic	□	It is necessary that ..
	◊	It is possible that …
	O	It is obligatory that …
Deontic Logic	***P***	It is permitted that …
	F	It is forbidden that …
Temporal Logic	***G***	It will always be the case that …
	F	It will be the case that …
	H	It has always been the case that …
	P	It was the case that …
Doxastic Logic	***Bx***	*x* believes that …

▶ http://plato.stanford.edu/entries/logic-modal/

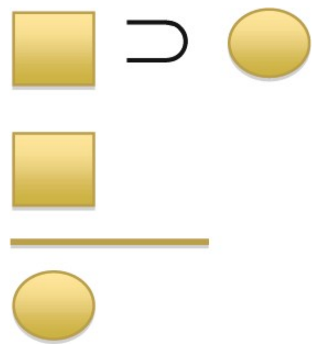
5.3.6 Visualizing the Rules of Inference

To be able to work proofs, you must be able to identify instances of the rules of inference. In the initial stage of learning this skill, you might find it helpful to use geometric shapes instead of the statement variables *p* and *q* to represent the form of a rule of inference.

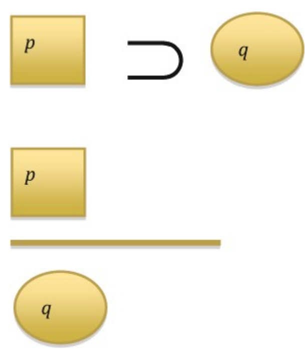
The form of *modus ponens* represented with variables is:

$$\begin{array}{l} p \supset q \\ \underline{p} \\ q \end{array}$$

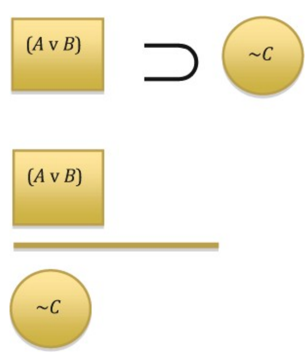
But note that you can represent *modus ponens* with geometric shapes like this:



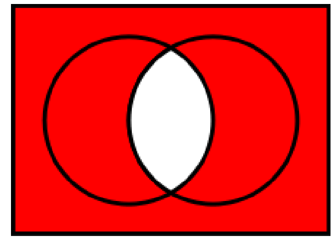
Now, in order to practice identifying the form, you can simply insert statements into the shapes. For example:



Any well-formed statement can go in the box or circle, and you will still have the form of *modus ponens*, no matter how complex the statement, as shown in the following example:



5.3.7 Sheffer stroke (NAND)



In Boolean functions and propositional calculus, the Sheffer stroke is written as "|" (vertical bar), "Dpq", or "↑" (an up arrow). This symbol denotes a logical operation that is equivalent to the negation of the conjunction operation, expressed in ordinary language as "not both".

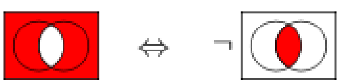
It is also called nand ("not and") or the alternative denial, since it says in effect that at least one of its operands is false. In Boolean algebra and digital electronics it is known as the NAND operation.

Like its dual, the NOR operator (also known as the Peirce arrow or Quine dagger), NAND can be used by itself, without any other logical operator, to constitute a logical formal system (making NAND functionally complete). This property makes the NAND gate crucial to modern digital electronics, including its use in computer processor design.

A	B	$A \vert B$
T	T	F
T	F	T
F	T	T
F	F	T

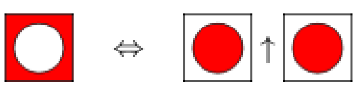
The Sheffer stroke ↑ is the negation of the conjunction:

$$P \uparrow Q \quad \Leftrightarrow \quad \neg(P \wedge Q)$$

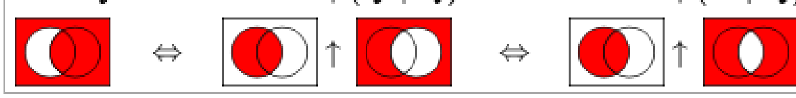


Expressed in terms of NAND ↑, the usual operators of propositional logic are:

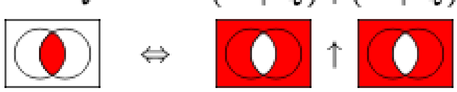
$$\neg P \quad \Leftrightarrow \quad P \uparrow P$$



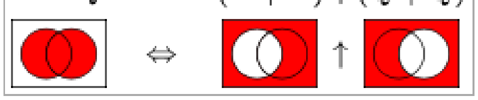
$$P \rightarrow Q \quad \Leftrightarrow \quad P \uparrow (Q \uparrow Q) \quad \Leftrightarrow \quad P \uparrow (P \uparrow Q)$$



$$P \wedge Q \quad \Leftrightarrow \quad (P \uparrow Q) \uparrow (P \uparrow Q)$$



$$P \vee Q \quad \Leftrightarrow \quad (P \uparrow P) \uparrow (Q \uparrow Q)$$



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